Lemons in Health Insurance

Jacob Kohlhepp

October 29, 2021

1 Lemons in Health Insurance: Akerloff Example

Note that Akerlof's lemons is essentially an issue of adverse selection, a term popularized by the insurance industry. Consider an applied example of Akerlof's lemons. Suppose there is a competitive market of health insurance companies. There is a single agent with privately known health status $\theta \sim F[\underline{\theta}, \bar{\theta}]$ and initial wealth w. The agent's health costs are ex-ante distributed $c(\theta) \sim N(\theta, \sigma^2)$. The agent has CARA utility given by $u(x) = -e^{-ax}$, $a > 0$. Suppose the firm proposes a premium and the agent can either accept the premium, go to another firm, or reject all offers and self-insure.

1.1 Solution

The agent's utility can be written in a certainty equivalent form given CARA utility and the normal lottery. Recall that we always offer full coverage, and utility from full coverage is just $-e^{-a(w-p)}$ where p is the premium and a is the coefficient of absolute risk aversion. However, agents will weigh this offer against self-insurance. Self-insurance means that the agent bears the risk completely. We can write utility in certainty equivalent form:

$$E[U(w - c(\theta))|\theta] = -exp(-a(w - \theta - a\frac{\sigma^2}{2}))$$

Thus for a premium p, an agent will opt in (equivalently, the IR constraint will hold) if:

$$-exp(-a(w - \theta - a\frac{\sigma^2}{2})) \leq -exp(-a(w - p)) \leftrightarrow w - \theta - a\frac{\sigma^2}{2} \geq w - p \implies p - a\frac{\sigma^2}{2} \leq \theta$$

This is clearly rising in θ. Call the threshold type where it binds $\hat{\theta}$. From perfect competition we know that the firms will bid up their coverage until they make zero profit. We can then apply Proposition 14 from Prof. Board's notes to say that the premium solves the zero profit condition:

$$p - E[\theta|p - a\frac{\sigma^2}{2} \leq \theta] = 0$$

Using Prof. Board's notation, we could define $r(\theta) := \theta + a\frac{\sigma^2}{2}$ and then the above could be re-written:

$$p - E[\theta|p \leq r(\theta)] = 0$$

We can also think of it as parameterized by the threshold type:

$$p - E[\theta|\theta \leq \hat{\theta}(p)] = 0$$

By the same argument on page 47 of the notes, there is a fixed point of this equation. However, in this case we have assumed normality and we can solve more explicitly. In particular, the conditional variable we are taking an expectation of is truncated normal, which has a nice closed form expression for the expectation (see Wikipedia); So we can write the fixed point equation as:

$$p - \left(E[\theta] - \sigma \frac{\phi((p - a\sigma^2/2 - E[\theta])/\sigma)}{1 - \Phi((p - a\sigma^2/2 - E[\theta])/\sigma)} \right) = 0$$

The third term is the inverse mills ratio, and it is increasing. As a result, the whole expression is increasing and there is a unique equilibrium premium price.

1. First, the outcome is not efficient, in the sense that ideally with $r > 0$ the firm should insure all agents, since there are always gains from trade. Yet in equilibrium we do not always have everyone trading. To have everyone trading, we need it to be true that $p - a\frac{\sigma^2}{2} \leq \underline{\theta}$. When a is low enough, this cannot be, as this implies negative profit.

2. Second, we can have full trade. In this case, we will have trade if either the risk is very high (σ is large) or people are very risk averse (a large). Intuitively, this makes the gains from trade large enough where the firm finds it optimal to insure all types.

3. Third, if there is zero risk aversion, $a = 0$, then $r(\theta) = \theta$ and there is no trade, as Prof. Board mentions. In this model this follows from the fact that there is no reason for trade: someone who is risk neutral does not want insurance.