Note on Multitasking with Harmful Effort

Jacob Kohlhepp

October 29, 2021

1 Model

1. Everyone is risk neutral.

2. We have two tasks, one productive which we denote 1 and one not productive which we denote 2. Effort done in each tasks is denoted by e_1, e_2.

3. The agent has outside option $\underline{U} = 0$ and cost of $c(e_1, e_2) = (e_1^2 + e_2^2)/2$.

4. The firm cares about output/revenue which is $R = 1 + e_1 - e_2$. It is important that one is minus - this is a case like Simon mentioned where a coefficient is negative.

5. We can only measure total effort, i.e. we see $m = e_1 + e_2$ so our contract can only be a linear function of effort: $w(e) = \alpha + \beta m$.

2 Agent's Problem - The IC Constraint

Agent solves the below (after deciding to take the contract):

$$\max_{e_1, e_2} \alpha + \beta m - c(e_1, e_2)$$

which, when simplified with things plugged in is:

$$\max_{e_1 e_2} \alpha + \beta(e_1 + e_2) - (e_1^2 + e_2^2)/2$$

FOCs:

$$\beta - e_1 = 0 \implies e_1 = \beta$$

$$\beta - e_2 = 0 \implies e_2 = \beta$$

These are the IC constraints. We see here that we can only choose effort combinations that are the same. From revenue, we know then that all incentive compatible efforts will result in $R(e) = 1$ for any α, β. So we might as well set $\beta = 0$. In this way we have already intuitively solved the problem. Let's pretend we aren't this clever, and just mechanically go through the steps.

2.1 Agent's Problem - The IR Constraint

We note that the agent only participates if the deal is weakly better than the outside option 0:

$$\alpha + \beta(e_1 + e_2) - (e_1^2 + e_2^2)/2 \geq 0$$

2.2 Firm's Problem

The firm then maximizes profit, which is:

$$\max_{\alpha,\beta,e_1,e_2} 1 + e_1 - e_2 - \alpha - \beta(e_1 + e_2)$$

subject to the agent participating:

$$\alpha + \beta(e_1 + e_2) - (e_1^2 + e_2^2)/2 \geq 0 \tag{IR}$$

and the agent having their incentives aligned:

$$e_1 = \beta\,,\ e_2 = \beta \tag{IC-FOCs}$$

We can argue that IR binds, because if it did not, the firm could lower α and strictly raise profit. Thus we set this as an equality and solve for α. Plugging in, we get the new objective:

$$\max_{\beta,e_1,e_2} 1 + e_1 - e_2 + \beta(e_1 + e_2) - (e_1^2 + e_2^2)/2 - \beta(e_1 + e_2)$$

simplifies to:

$$\max_{\beta,e_1,e_2} 1 + e_1 - e_2 - (e_1^2 + e_2^2)/2$$

Note that this is surplus: it is the of effort (revenue) less the cost of this effort. Transfers wash out. To finish solving, plug in the IC-FOCs:

$$\max_{\beta} 1 + \beta - \beta + \beta(\beta + \beta) - (\beta^2 + \beta^2)/2 - \beta(\beta + \beta)$$

which simplifies to:

$$\max_{\beta} 1 - \beta^2$$

Clearly this is minimized at $\beta = 0$. α can be derived from the binding IR constraint:

$$\alpha = \beta(e_1 + e_2) + (e_1^2 + e_2^2)/2 = 0$$

The agent needs no payment because we require no effort from him/her.