# The Inner Beauty of Firms

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# A Tale of Two Salons

## Westwood Barber Shop


12/10/2014 · Updated review

A lovely stylist named Minoo did an incredible job. She colored my hair, freshened up my bob and gave me a great blow dry. The prices are unbelievable, 25 for color, 20 for haircut and 20 for blow dry.


3/10/2019

Throughly enjoyable quality cut from the delightful owners of the salon. At 81 she cut while he cleaned.

## John Frieda Salon


6/9/2011

In addition to seeing a different person for your cut and color all the stylists have assistants and they are usually the ones that take you back for washing and drying if your stylist is busy. I've had days where I swear 4-5 people worked on me like I'm a celebrity or something, which speaking of there are often quite a few getting their hair done as well.


1/23/2013

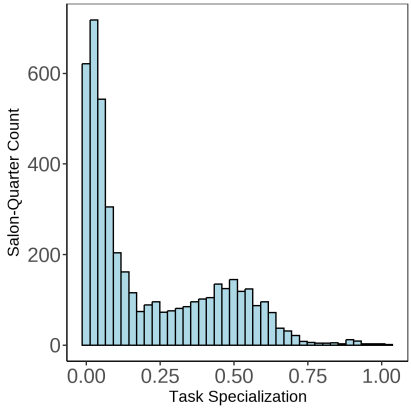
A cut and color here costs more than a monthly payment for some cars.

**Source:** Yelp.com. Review text truncated for brevity.

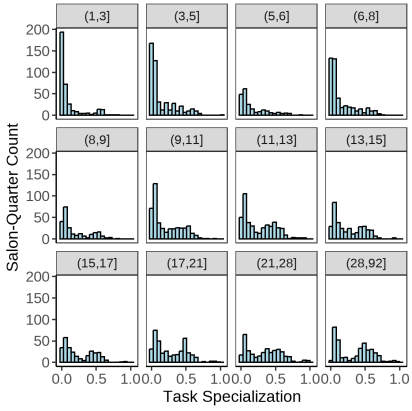
# A Tale of Two Salons: A Representative Anecdote

| Statistic | N | Mean | Min | Pctl(25) | Median | Pctl(75) | Max |
|---|---|---|---|---|---|---|---|
| Labor Productivity | 4,599 | 1.81 | 0.003 | 1.03 | 1.38 | 2.05 | 42.80 |
| S-index | 4,599 | 0.22 | 0.00 | 0.03 | 0.11 | 0.41 | 1.02 |

# A Tale of Two Salons: More Than Scale




(a) All Salon Quarters




(b) By Number of Employees

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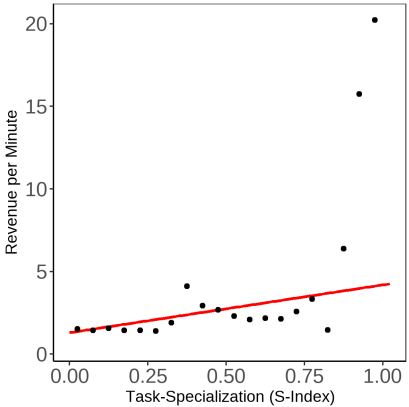


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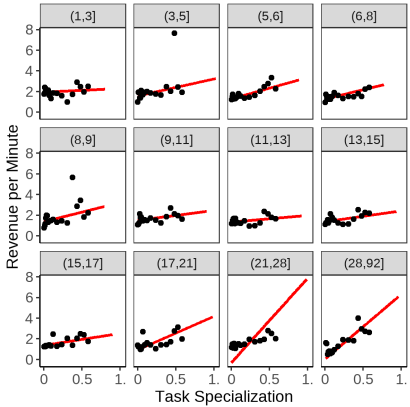
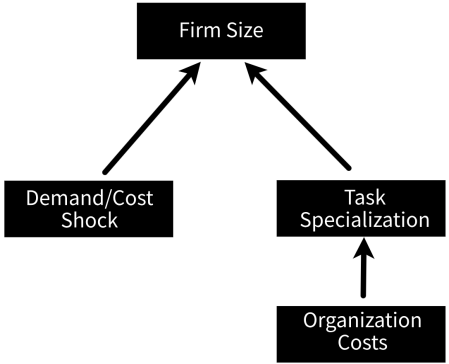
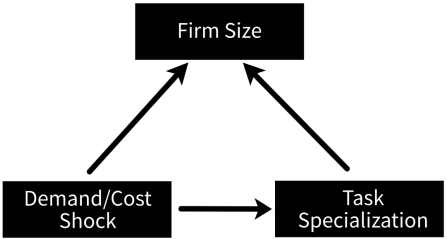



(b) By Number of Employees

# Causal Relationships Between Firm-Size and Specialization




(a) This Model




(b) Others

# Research Question

How are differences in establishment internal organization related to establishment and aggregate productivity?

# This Paper

1. Internal task specialization is dispersed and associated with productivity.
   - Management software: 10.8 million task assignments from competing salons.

2. Salons with different coordination costs choose how to assign tasks in labor and product market equilibrium.
   - KL divergence coordination costs provide tractable theoretical properties.
   - Coordination costs, worker skills, and wages are identified from task assignments.

3. Internal reorganization shapes equilibrium responses to counterfactual shocks.
   - Two workers can be substitutes at one salon and complements at another.
   - Low-wage immigration: productivity $-1.0\%$ without reorg, $+1.4\%$ with reorg.

# Contributions to Literature

## Endogenous and Firm-Specific Task Specialization

Lazear 2009 (task-mix); Haanwinckel 2023 (multi-worker firms); Garicano 2000 (vertical workers); Adenbaum 2022 (org. costs); Lindenlaub 2017 (multi-skill workers); Baker, Gibbons, and Murphy 2002 (relational contracts); Garicano and Wu 2012 (knowledge); Meier, Stephenson, and Perkowski 2019 (trust); Martinez et al. 2015 (culture); Alchian and Demsetz 1972, Baker and Hubbard 2003 (monitoring)

## Task Assignment as a Determinant of Productivity Dispersion

Bassi et al. 2023 (across firms); Minni 2023 (across managers); Bloom and Van Reenen 2007 (management); Syverson 2011 (survey across fields); Kuhn et al. 2023

## Estimation of Task-Based Production Functions

Key features: no wage data, multi-dim. workers, not Hicks neutral

Caliendo et al. 2012 (vertical wage-based approach); Berry, Levinsohn, and Pakes 1995 (demand + firm conduct); Caplin and Nalebuff (1991) (uniqueness); Matêjka and McKay 2015 (key tool); Rubens 2023 (non-Hicks neutral example)

# Table of Contents

# What is an Organization?

## Definition

*A firm's organization ($B_j$) is a matrix where element $(m, k)$ is the fraction of labor assigned to worker $m$ and task $k$.*

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Tasks

| | Cut | Color | Dry | |
|---|---|---|---|---|
| A | .1 | .2 | .1 | .4 |
| B | .1 | .1 | .1 | .3 |
| C | .2 | .05 | .05 | .3 |
| Tot. | .4 | .35 | .25 | |

Labor Demand ($E$)

Task-Mix ($\alpha$)

# Measuring Internal Task-Specialization

Suppose we observe this organization:

| | Tasks | | | |
|---|---|---|---|---|
| | Cut | Color | Dry | |
| A | .1 | .2 | .1 | .4 |
| B | .1 | .1 | .1 | .3 |
| C | .2 | .05 | .05 | .3 |
| Tot. | .4 | .35 | .25 | |

Task-Mix ($\alpha$)

Labor Demand ($E$)

# Measuring Internal Task-Specialization

Construct a generalist benchmark ($G(B)$):

Tasks

| | Cut | Color | Dry | |
|---|---|---|---|---|
| A | .1 | .2 | .1 | .4 |
| B | .1 | .1 | .1 | .3 |
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| Tot. | .4 | .35 | .25 | |

Task-Mix ($\alpha$)

Tasks

| | Cut | Color | Dry | |
|---|---|---|---|---|
| A | | | | |
| B | | | | |
| C | | | | |
| Tot. | | | | |

Labor Demand ($E$)

# Measuring Internal Task-Specialization

Hold fix what needs to be done (task-mix):

| | Tasks | | | |
|---|---|---|---|---|
| | Cut | Color | Dry | |
| A | .1 | .2 | .1 | .4 |
| B | .1 | .1 | .1 | .3 |
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Task-Mix ($\alpha$)

| | Tasks | | | |
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| Tot. | .4 | .35 | .25 | |

Labor Demand ($E$)

# Measuring Internal Task-Specialization

Hold fix who is employed (Labor Demand):

Tasks

| | Cut | Color | Dry | |
|---|---|---|---|---|
| A | .1 | .2 | .1 | .4 |
| B | .1 | .1 | .1 | .3 |
| C | .2 | .05 | .05 | .3 |
| Tot. | .4 | .35 | .25 | |

Tasks

| | Cut | Color | Dry | |
|---|---|---|---|---|
| A | | | | .4 |
| B | | | | .3 |
| C | | | | .3 |
| Tot. | .4 | .35 | .25 | |

Labor Demand ($E$)

Task-Mix ($\alpha$)

# Measuring Internal Task-Specialization

Randomly assign workers to tasks ($G(B_j)(i,k) = E_i \cdot \alpha_k$)

Tasks

| | Cut | Color | Dry | |
|---|---|---|---|---|
| A | .1 | .2 | .1 | .4 |
| B | .1 | .1 | .1 | .3 |
| C | .2 | .05 | .05 | .3 |
| Tot. | .4 | .35 | .25 | |

Task-Mix ($\alpha$)

Tasks

| | Cut | Color | Dry | |
|---|---|---|---|---|
| A | .16 | .14 | .1 | .4 |
| B | .12 | .105 | .075 | .3 |
| C | .12 | .105 | .075 | .3 |
| Tot. | .4 | .35 | .25 | |

Labor Demand ($E$)

# The S-index

A firm is task-specialized if it is "far" from the counterfactual generalist firm.

## Definition 1

The task-specialization index **(s-index)** of a firm with org. structure $B$ is given by:

$$I(B) := \underbrace{D_{KL}(B|G(B))}_{\text{Kullback-Leibler divergence}} = \sum_{m,k} B(m,k) log\left(\frac{B(m,k)}{G(B)(m,k)}\right)$$

# The S-index

A firm is task-specialized if it is "far" from the counterfactual generalist firm.

## Definition 2

The task specialization index **(s-index)** of a firm with org. structure $B$ is given by:

$$I(B) := \underbrace{D_{KL}(B|G(B))}_{\text{Kullback-Leibler divergence}} = \sum_{m,k} B(m,k) log\left(\frac{B(m,k)}{\underbrace{\alpha_k}_{\text{task-mix}} \cdot \underbrace{E_m}_{\text{labor demand}}}\right)$$

# Table of Contents

# Model

## Firms: $j = 1, ...., J$

- Firm $j$ has a coordination cost parameter $\gamma_j$ (not Hicks neutral)
- Firm $j$ requires $\bar{a}_j$ labor and must assign a fraction $\alpha_j(k)$ to task $k$
- Firm $j$ has a constant marginal cost: $\alpha_j \cdot c + \omega_j$ (material cost + Hicks neutral )

## Workers: $m = 1, ...., M$

- Skill level $\bar{\theta}_m \in \mathbb{R}$, skill set $\theta_m \in \mathbb{R}^K$ and labor supply $l_m \in \mathbb{R}_+$
- Worker $m$ performs task $k$ with quality $\bar{\theta}_m + \theta_m(k)$
- Worker-specific wages $w \in \mathbb{R}_+^M$

# Model

## Firm Actions

(simultaneously chosen)

- Price $p_j \in \mathbb{R}_+$ (Bertrand-style)
- Relative Labor demand $E_j \in \mathbb{R}_+^M$ (fraction of work done by each worker)
- Task assignment $b_j \in \mathbb{R}_+^M \times \mathbb{R}_+^K$ (how each worker splits their time)

## Per Unit Coordination Costs

- Sum of the KL divergence of each worker's assignment from the firm's task-mix weighted by relative labor demand: $\gamma_j \cdot \sum_m E_j(m) D_{KL}(b_j(m, \cdot) || \alpha_j)$
- This is exactly $\gamma_j$ times the s-index.

# Model

## Product Market

- Consumers observe task assignments and prices and purchase based on utility $u_{z,j} = \xi_j + \nu_j - \rho p_j + \epsilon_{z,j}$ with $\epsilon_{z,j}$ i.i.d. Type-1 EV (no purchase normalized to $\epsilon_{z,0}$)
- $\xi_j$ is average quality across all workers and tasks given $b_j, E_j$

## Equilibrium

- Firm strategies $\{p_j, E_j, b_j\}_{j=1}^{J}$ are a Nash Equilibrium under wage $w$.
- Call this a fixed $w$-subgame.
- Wages $w$ such that strategies in the fixed-$w$ subgame clear the labor market.

# Table of Contents

# Understanding Firm Strategies

- The only worker characteristic that impacts firm profit is worker skill set ($\theta_m$).
  - Vertical skill level wages always satisfy no arbitrage condition.
- Firm internal organization is separable from pricing strategy.
- Internal organization depends only on wages.
- Prices depend on own and competitor prices, own and competitor internal organization, and wages.

# Nash Equilibrium Uniqueness for Fixed Wages

## Proposition

*If there exists a positive semi-definite $N \times K$ matrix with no duplicate rows which contains all skill set vectors as rows, then there exists a unique Nash equilibrium in prices ($p_j$), task assignments ($b_j$) and relative labor demands ($E_j$) for every fixed-wage subgame.*

- Proof Sketch Part 1: Pricing-internal organization separability.
- Proof Sketch Part 2: Given separability, internal organization equivalent to rational inattention problem, pricing is Bertrand-logit.
- For any given wage vector there is at most one equilibrium.
- Workers can be complements or substitutes at different firms, thus full uniqueness is unclear.

# Firm and Worker Specific Task Assignments

## Theorem

*The profit-maximizing task assignment for a worker with skill set $i$ at firm $j$ is given by*

$$b_j(i,k) = \alpha_j(k)\frac{exp[\gamma_j^{-1}(\rho^{-1}\theta_i(k) - w(i))]}{\sum_{i'} E_j(i')exp[\gamma^{-1}(\rho^{-1}\theta_{i'}(k) - w(i'))]},$$

*and it satisfies the following properties:*

1. ***Relative Law of Demand:*** *As $w(i)$ increases, skill set $i$'s share of labor at firm $j$ ($E_j(i)$) decreases.*
2. ***Incomplete Specialization:*** *All workers employed by firm $j$ ($E_j(i) > 0$) spend a strictly positive amount of time on all tasks performed at the firm ($\{k|\alpha_j(k) > 0\}$).*
3. ***Maximum Coworker Diversity:*** *Either the number of skill sets employed at a firm is less than or equal to the number of tasks, or there exists another profit-maximizing task assignment strategy where this is true.*

# Table of Contents

# Key Identification Challenge

We observe task assignments $(b_j(m,k))$ to individual workers, but...

$$b_j(m,k) = \alpha_j(k)\frac{exp[\gamma_j^{-1}(\rho^{-1}\theta_{i_m}(k) - w(i_m))]}{\sum_{i'} E_j(i')exp[\gamma^{-1}(\rho^{-1}\theta_{i'}(k) - w(i'))]}$$

- Salon task-based production functions differ.

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- Salon task-based production functions differ.
- Salon coordination costs differ.

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- Salon task-based production functions differ.
- Salon coordination costs differ.
- The skill set of each worker ($i_m$) is unobserved.

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- Salon task-based production functions differ.
- Salon coordination costs differ.
- The skill set of each worker ($i_m$) is unobserved.
- One worker's tasks depends on the skills/tasks of all their coworkers.

# Identification

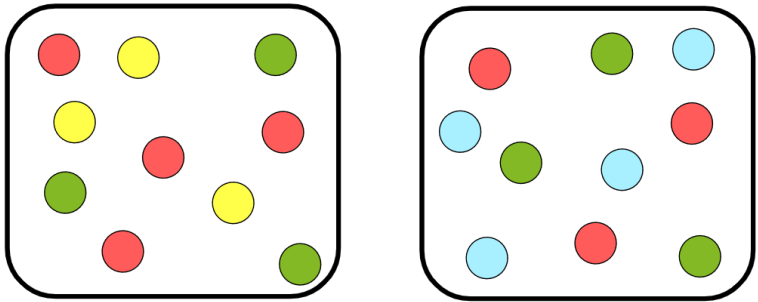
Assume we only observe task assignments and firm market-shares and prices.

## Theorem

*Suppose the set of wage-adjusted skill vectors $\{\theta_i - \rho w_i e\}_{i=1}^{N}$ is linearly independent. Wages ($w$), price sensitivity ($\rho$), material costs ($m$) and the skill set parameters ($\theta$) are identified. The coordination cost parameters ($\gamma_j$) and the skill sets of all workers ($\{\theta_m\}_{m=1}^{M}$) at firms with a strictly positive s-index ($I_j > 0$) are identified. Lower bounds on the organization cost parameters of firms with an s-index of 0 are identified.*

- Two firms are connected if they employ 2 or more of the same skill sets.
- Identification requires (1) sufficient differences in wages and skills (2) a connected set of firms that collectively employ all worker skill sets.
- Sufficient condition: one firm employs all worker skill sets.

# Grouping Workers By Skill Set Within Firm



- Circles are workers (indexed by $m$), boxes are salons (indexed by $j$)
- Colors are unobserved skill set groups of workers, denoted $i_m$
- We observe only task assignments.

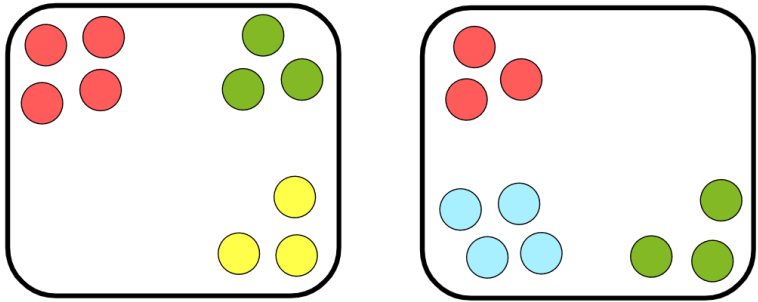
# Grouping Workers By Skill Set Within Firm

- Worker 1 in firm $j$ has task assignment:

$$b_j(1,k) = \alpha_j(k)\frac{exp[\gamma_j^{-1}(\rho^{-1}\theta_{i_1}(k) - w(i_1))]}{\sum_{i'} E_j(i')exp[\gamma^{-1}(\rho^{-1}\theta_{i'}(k) - w(i'))]}$$

- If 2 workers are at the same firm, they have the same task assignment if and only if they have the same skill set.

# Grouping Workers By Skill Set Within Firm



- Within firm, workers are now grouped correctly.
- But not across firms.

# Groupings Workers by Skill Sets Across Firms

- Org. cost and task-mix confound task assignments across firms.

$$b_j(m,k) = \alpha_j(k)\frac{exp[\gamma_j^{-1}(\rho^{-1}\theta_{i_m}(k) - w(i_m))]}{\sum_{i'} E_j(i')exp[\gamma^{-1}(\rho^{-1}\theta_{i'}(k) - w(i'))]}$$

-

# Groupings Workers by Skill Sets Across Firms

- Differences in org. cost and task-mix confound grouping across firms.
- Take another worker at firm $j$ but with a different skill set. Call them worker 2:

$$b_j(i_2, k) = \alpha_j(k) \frac{exp(-\gamma_j^{-1} w(i_2) + (\rho\gamma_j)^{-1}\theta_{i_2}(k)))}{\sum_{i'} E_j(i') exp(-\gamma_j^{-1} w(i') + (\rho\gamma)^{-1}\theta_{i'}(k))}$$

# Groupings Workers by Skill Sets Across Firms

- Differences in org. cost and task-mix confound grouping across firms.

- Divide the task assignment of worker 1 by that of worker 2 across all tasks:

$$\frac{b_j(i_1,k)}{b_j(i_2,k)} = \frac{\alpha_j(k)\frac{exp(-\gamma_j^{-1}w(i_1)+(\rho\gamma_j)^{-1}\theta_{t_1}(k)))}{\sum_{i'} E_j(i')exp(-\gamma_j^{-1}w(i')+(\rho\gamma)^{-1}\theta_{i'}(k))}}{\alpha_j(k)\frac{exp(-\gamma_j^{-1}w(i_2)+(\rho\gamma_j)^{-1}\theta_{i_2}(k)))}{\sum_{i'} E_j(i')exp(-\gamma_j^{-1}w(i')+(\rho\gamma)^{-1}\theta_{i'}(k))}}$$

# Groupings Workers by Skill Sets Across Firms

- Differences in org. cost and task-mix confound grouping across firms.
- This removes most of the firm-level confounding:

$$\frac{b_j(t_1,k)}{b_j(t_2,k)} = \frac{exp(-\gamma_j^{-1}w(i_1) + (\rho\gamma_j)^{-1}\theta_{i_1}(k)))}{exp(-\gamma_j^{-1}w(i_2) + (\rho\gamma_j)^{-1}\theta_{i_2}(k)))}$$

# Groupings Workers by Skill Sets Across Firms

- Differences in org. cost and task-mix confound grouping across firms.
- Take logs:

$$log\left(\frac{b_j(i_1,k)}{b_j(i_2,k)}\right) = (\rho\gamma_j)^{-1}\left([\theta_{i_1}(k) - \rho w(i_1)] - [\theta_{i_2}(k) - \rho w(i_2)]\right)$$

# Groupings Workers by Skill Sets Across Firms

- Differences in org. cost and task-mix confound grouping across firms.

- Divide the vector by its Euclidean norm:

$$\frac{log\left(\frac{b_j(i_1,k)}{b_j(i_2,k)}\right)}{\left|\left\{log\left(\frac{b_j(i_1,k')}{b_j(i_2,k')}\right)\right\}_{k'=1}^{K}\right|} = \frac{(\rho\gamma_j)^{-1}\Big(\theta_{i_1}(k) - \rho w(i_1) - [\theta_{i_2}(k) - \rho w(i_2)]\Big)}{\left(\sum_{k'}\left[(\rho\gamma_j)^{-1}(\theta_{i_1}(k') - \rho w(i_1) - [\theta_{i_2}(k') - \rho w(i_2)])\right]^2\right)^{1/2}}$$

# Groupings Workers by Skill Sets Across Firms

- Differences in org. cost and task-mix confound grouping across firms.

- This removes the org. cost parameter:

$$\frac{log\left(\frac{b_j(i_1,k)}{b_j(i_2,k)}\right)}{\left|\left\{log\left(\frac{b_j(i_1,k')}{b_j(i_2,k')}\right)\right\}_{k'=1}^{K}\right|} = \frac{\left(\theta_{i_1}(k) - \rho w(i_1) - [\theta_{i_2}(k) - \rho w(i_2)]\right)}{\left(\sum_{k'}\left[(\theta_{i_1}(k') - \rho w(i_1) - [\theta_{i_2}(k') - \rho w(i_2)])\right]^2\right)^{1/2}}$$

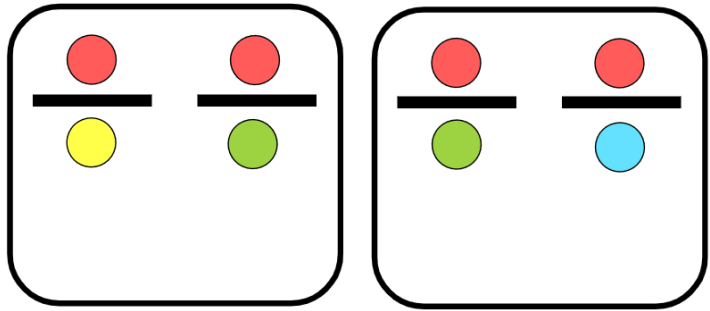
# Groupings Workers by Skill Sets Across Firms

- Differences in org. cost and task-mix confound grouping across firms.
- Call these coworker log-ratio vectors.

$$\frac{log\left(\frac{b_j(i_1,k)}{b_j(i_2,k)}\right)}{\left|\left\{log\left(\frac{b_j(i_1,k')}{b_j(i_2,k')}\right)\right\}_{k'=1}^{K}\right|} = \frac{\left(\theta_{i_1}(k) - \rho w(i_1) - [\theta_{i_2}(k) - \rho w(i_2)]\right)}{\left(\sum_{k'}\left[(\theta_{i_1}(k') - \rho w(i_1) - [\theta_{i_2}(k') - \rho w(i_2)])\right]^2\right)^{1/2}}$$

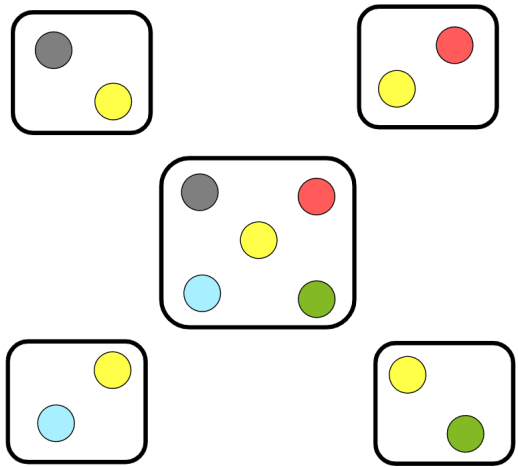
- If a firm employs 5 skill sets, each worker has 4 coworker log ratio vectors.

# Grouping Workers By Skill Set Across Firms



- Compare the coworker log ratios. They will match if and only if the numerator workers AND denominator workers have the same skill set.
- If firms are sufficiently connected in terms of pairs of skill sets, this can be repeated to classify all workers.

# Sufficient Condition: One Firm Has All 5 Skill Sets



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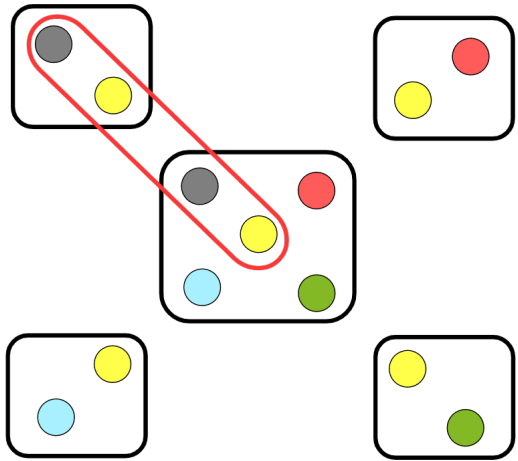

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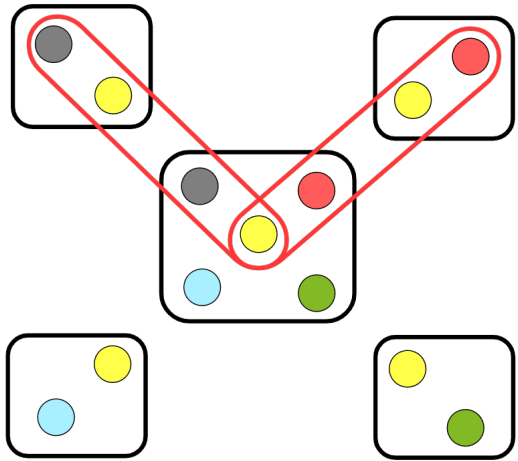

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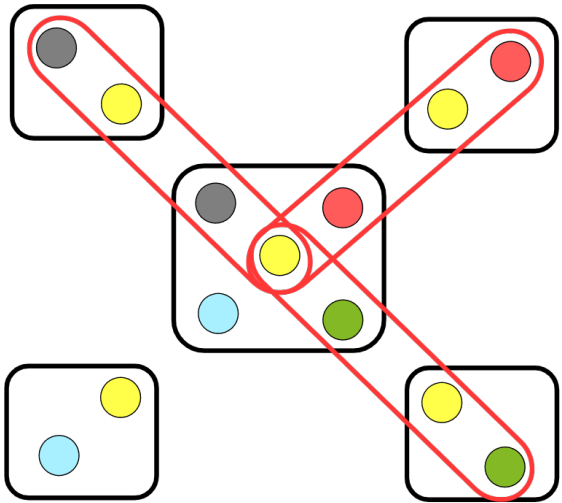

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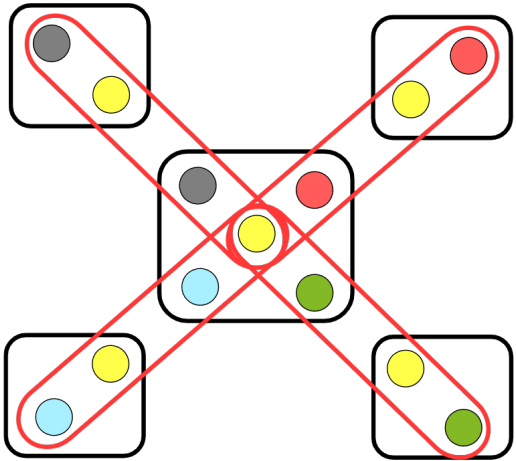
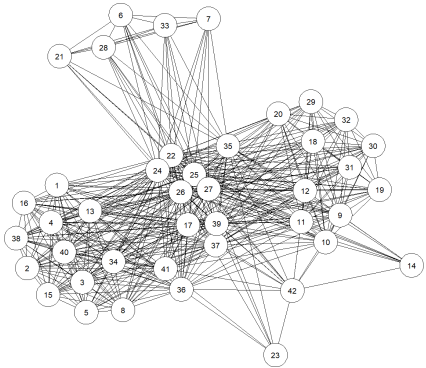
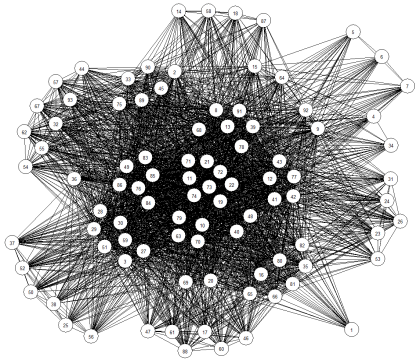


Could impose other conditions (pigeon hole principle).

# A Network of Shared Skill Sets

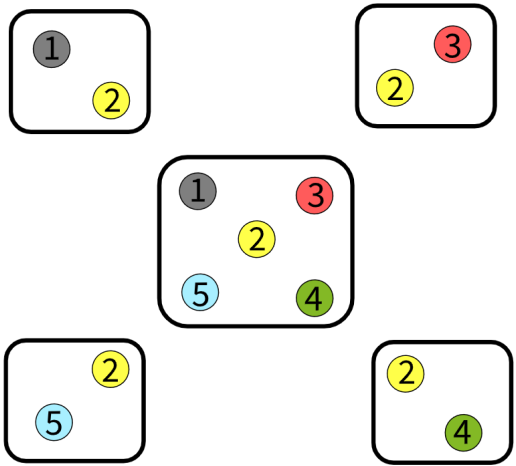



Manhattan (2019 Q1-Q4)



Los Angeles (2019 Q1-Q4)

# Worker Skill Set Categories are Identified



# Estimation Procedure Sketch

1. **Within-salon classification:** Group workers into unique skill sets within salon.

2. **Market-wide classification:** Compare coworker pairs to classify workers into skill sets across firms.
   - Done by finding a reference salon that employs all skill sets.
   - After this step, $\{b_j, E_j\}$ are known.

3. **GMM.** With skill sets known, estimation involves two linear systems of moments.
   - Estimation procedure implemented in paper is slightly more complicated.

# Table of Contents

# Los Angeles County Wages and Skills

| Skill Set | Wage | Admin. | Blowdry/Style | Color/Etc. | Haircut/Shave | Nail/Misc. |
|---|---|---|---|---|---|---|
| 1 | - | -0.028 | -0.275 | 0.876 | -5.248 | -61.626 |
| | - | ( 4.874) | ( 2.737) | ( 1.175) | ( 1.509) | ( 29.540) |
| 2 | 536.753 | -5.466 | 13.326 | 2.332 | -6.157 | -9.492 |
| | ( 210.962) | ( 3.919) | ( 10.040) | ( 1.968) | ( 2.535) | ( 2.699) |
| 3 | -7.202 | 0.043 | 1.570 | -0.439 | -3.733 | -6.118 |
| | ( 24.149) | ( 1.343) | ( 2.155) | ( .965) | ( .701) | ( 10.649) |
| 4 | 20.981 | -0.305 | 3.759 | 0.751 | -5.383 | -3.982 |
| | ( 33.875) | ( .954) | ( 2.710) | ( 1.231) | ( 1.351) | ( 2.395) |
| 5 | 59.820 | 0.946 | -2.708 | 1.654 | -3.703 | -3.676 |
| | ( 33.640) | ( 1.662) | ( 1.189) | ( 1.108) | ( 1.232) | ( 1.419) |

- High wage, high skill color/style specialist
-
-

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- High wage, high skill color/style specialist
- Medium wage, medium skill color/haircut specialist
-

# Los Angeles County Wages and Skills

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- High wage, high skill color/style specialist
- Medium wage, medium skill color/haircut specialist
- Low wage, low skill generalists

# Table of Contents

# Labor-Labor Substitution Patterns (Los Angeles)

| | Skill Set 1 | | | Skill Set 2 | | | Skill Set 3 | | | Skill Set 4 | | | Skill Set 5 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Skill Set | Max. | Med. | Min. | Max. | Med. | Min. | Max. | Med. | Min. | Max. | Med. | Min. | Max. | Med. | Min. |
| 1 | 0.00 | 0.00 | -0.52 | 0.05 | 0.00 | -0.35 | 0.31 | 0.00 | -0.16 | 0.33 | 0.00 | -0.21 | 0.38 | 0.00 | 0.00 |
| 2 | 0.11 | 0.00 | -0.18 | 0.00 | 0.00 | -0.27 | 0.15 | 0.00 | -0.20 | 0.18 | 0.00 | 0.00 | 0.00 | 0.00 | -0.13 |
| 3 | 0.04 | 0.00 | -0.15 | 0.10 | 0.00 | -0.22 | 0.00 | 0.00 | -0.22 | 0.14 | 0.00 | -0.10 | 0.11 | 0.00 | 0.00 |
| 4 | 0.55 | 0.00 | -0.33 | 0.52 | 0.00 | 0.00 | 0.42 | 0.00 | -0.45 | 0.00 | 0.00 | -0.52 | 0.41 | 0.00 | -0.02 |
| 5 | 0.80 | 0.00 | 0.00 | 0.00 | 0.00 | -0.39 | 0.48 | 0.00 | 0.00 | 0.54 | 0.00 | -0.04 | 0.00 | 0.00 | -0.61 |

# Labor-Labor Substitution Patterns (Los Angeles)

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| 2 | 0.11 | 0.00 | -0.18 | 0.00 | 0.00 | -0.27 | 0.15 | 0.00 | -0.20 | 0.18 | 0.00 | 0.00 | 0.00 | 0.00 | -0.13 |
| 3 | 0.04 | 0.00 | -0.15 | 0.10 | 0.00 | -0.22 | 0.00 | 0.00 | -0.22 | 0.14 | 0.00 | -0.10 | 0.11 | 0.00 | 0.00 |
| 4 | 0.55 | 0.00 | -0.33 | 0.52 | 0.00 | 0.00 | 0.42 | 0.00 | -0.45 | 0.00 | 0.00 | -0.52 | 0.41 | 0.00 | -0.02 |
| 5 | 0.80 | 0.00 | 0.00 | 0.00 | 0.00 | -0.39 | 0.48 | 0.00 | 0.00 | 0.54 | 0.00 | -0.04 | 0.00 | 0.00 | -0.61 |

- Recall: without org. costs, workers are perfect substitutes.
- Even with fixed task intensities in the same market, two workers can be complements at one firm and substitutes at another.
- In full equilibrium, shocks to one type of worker can have widely different implications for different firms.
- For workers, the impacts of a shock are NOT tied to their position in the original wage distribution.

# Labor-Labor Substitution Patterns (Los Angeles)

| | Skill Set 1 | | | Skill Set 2 | | | Skill Set 3 | | | Skill Set 4 | | | Skill Set 5 | | |
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| 1 | 0.00 | 0.00 | -0.52 | 0.05 | 0.00 | -0.35 | 0.31 | 0.00 | -0.16 | 0.33 | 0.00 | -0.21 | 0.38 | 0.00 | 0.00 |
| 2 | 0.11 | 0.00 | -0.18 | 0.00 | 0.00 | -0.27 | 0.15 | 0.00 | -0.20 | 0.18 | 0.00 | 0.00 | 0.00 | 0.00 | -0.13 |
| 3 | 0.04 | 0.00 | -0.15 | 0.10 | 0.00 | -0.22 | 0.00 | 0.00 | -0.22 | 0.14 | 0.00 | -0.10 | 0.11 | 0.00 | 0.00 |
| 4 | 0.55 | 0.00 | -0.33 | 0.52 | 0.00 | 0.00 | 0.42 | 0.00 | -0.45 | 0.00 | 0.00 | -0.52 | 0.41 | 0.00 | -0.02 |
| 5 | 0.80 | 0.00 | 0.00 | 0.00 | 0.00 | -0.39 | 0.48 | 0.00 | 0.00 | 0.54 | 0.00 | -0.04 | 0.00 | 0.00 | -0.61 |

- Recall: without org. costs, workers are perfect substitutes.
- Even with fixed task intensities in the same market, two workers can be complements at one firm and substitutes at another.
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- For workers, the impacts of a shock are NOT tied to their position in the original wage distribution.

# Table of Contents

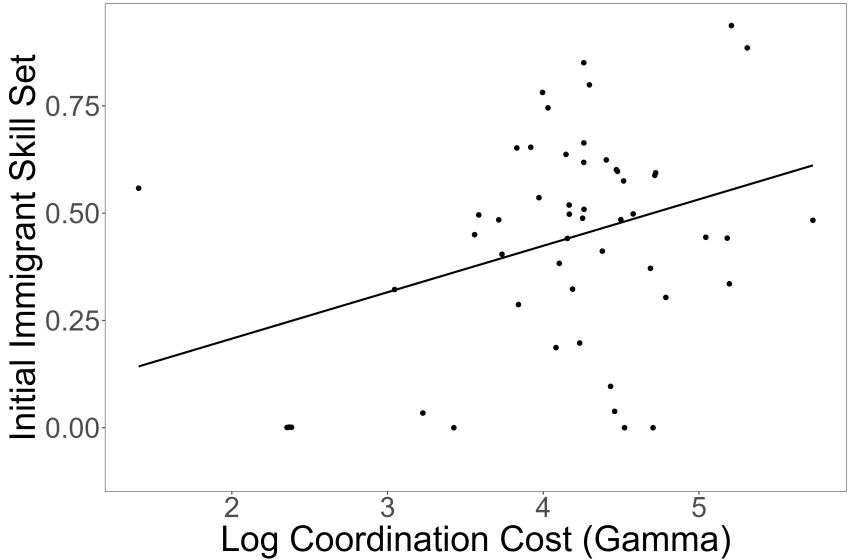
# Industry Equilibrium Counterfactuals

- Reallocation equilibrium: allow firms to adjust prices, but not relative labor demand and task assignments.
  - This captures how heterogeneity in initial internal organization reallocates labor.
  - Reallocation effects exist in most heterogeneous firm models.
- Reorganization (full) equilibrium: adjustment of prices, relative labor demand and task assignments.
  - This captures how reorganization within the firm impacts aggregate productivity.
  - Allowing firms to differ in their reorganization is novel.
- Main Outcome: Labor productivity, defined as total endogenous quality delivered divided by total labor.

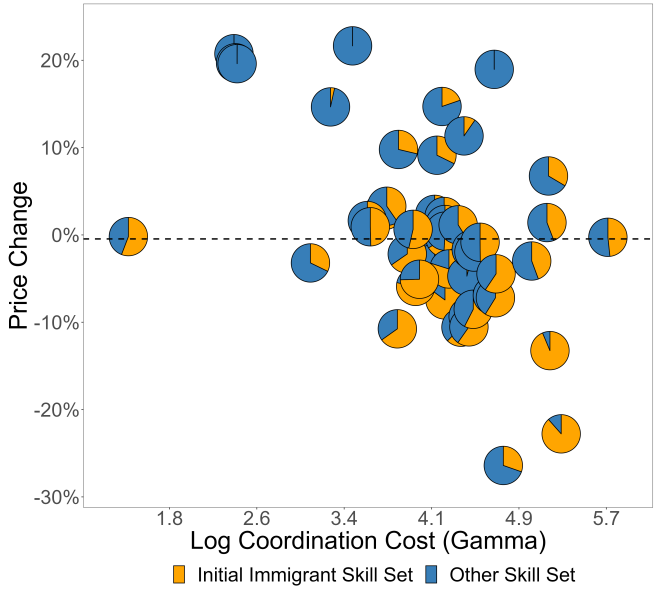
# Industry Equilibrium Counterfactuals

- **Immigration.** 10% increase in the total labor supply of the lowest wage skill set.
  - Focus on Los Angeles County, but Cook County and Manhattan in paper.
- **Sales Tax Increase.** 4 percentage point increase of the tax on salon services.
- **Management Diffusion.** Each salon learns and then adopts the management practices of the next best salon.
- **Increase in Market Concentration.** Half of the salons in each market are removed.

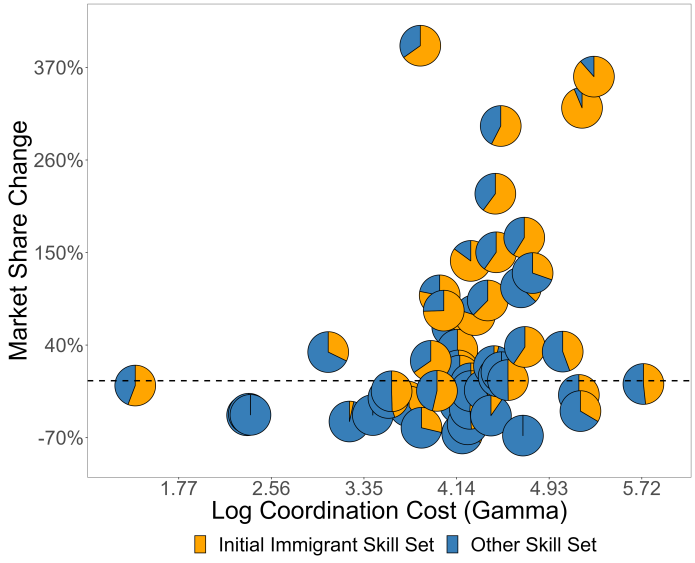
# Reallocation Eqm: Initial Employment of Immigrant Skill Set



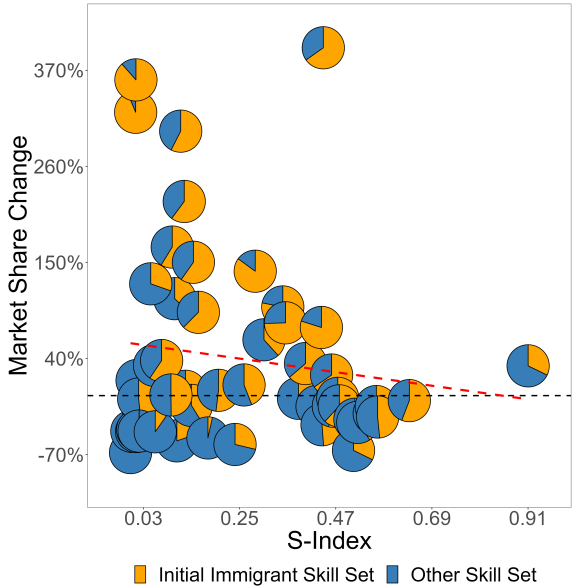
# Reallocation Eqm: High Coordination Cost Salons Reduce Prices



# Reallocation Eqm: High Coordination Cost Salons Gain Market Share



# Reallocation Eqm: On Net Specialized Jobs are Lost

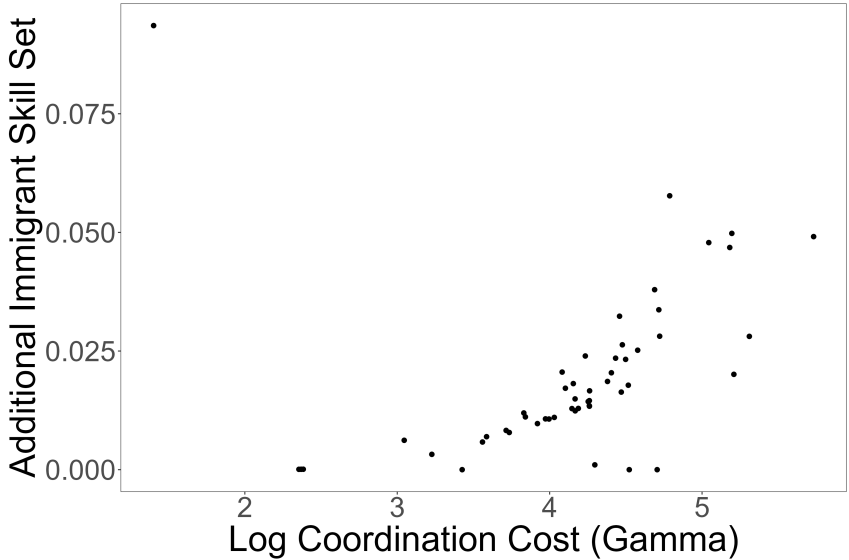


# Reallocation Equilibrium

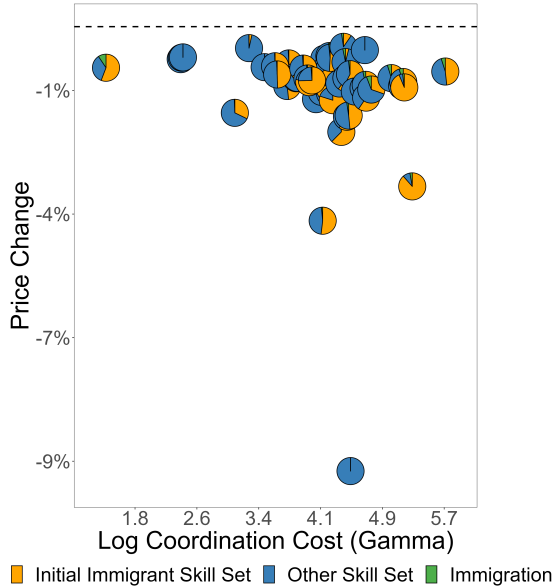
Labor-Weighted Average S-Index: $-1.4\%$

Labor Productivity: $-1.0\%$

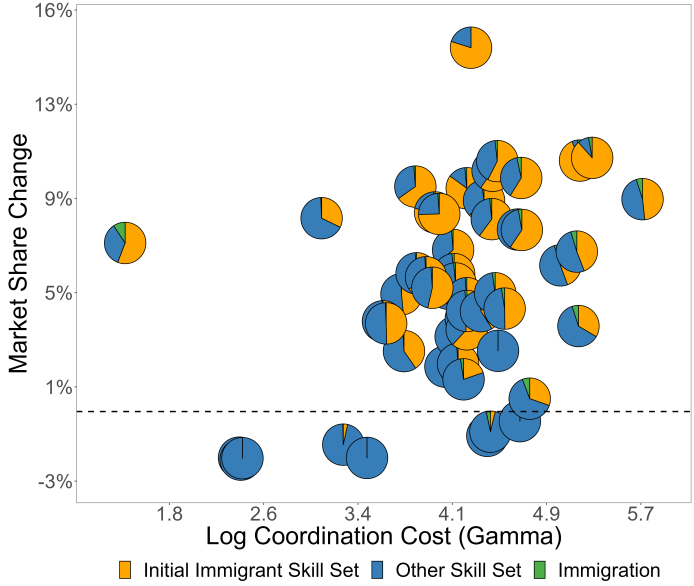
# Reorganization Eqm: Most Salons Incorporate Immigrant Skill Set



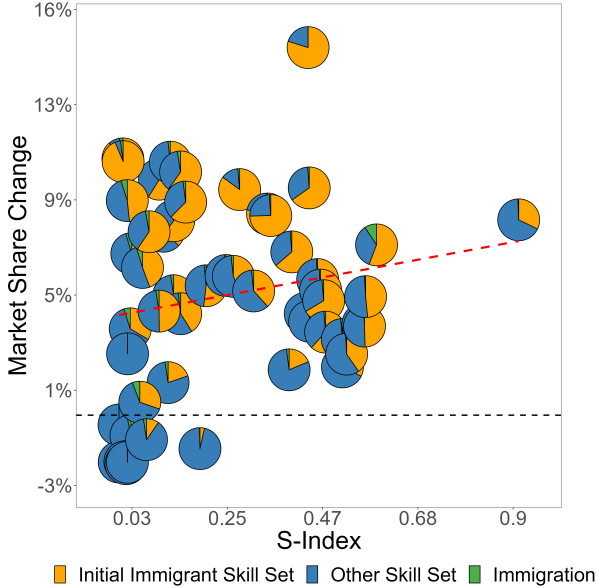
# Reorganization Eqm: All Salons Reduce Prices



# Reorganization Eqm: Most Salons Increase Market Share



# Reorganization Eqm: On Net Specialized Jobs are Created



# Reorganization Equilibrium

Labor-Weighted Average S-Index: $+0.4\%$

Labor Productivity: $+1.4\%$

Thank You!

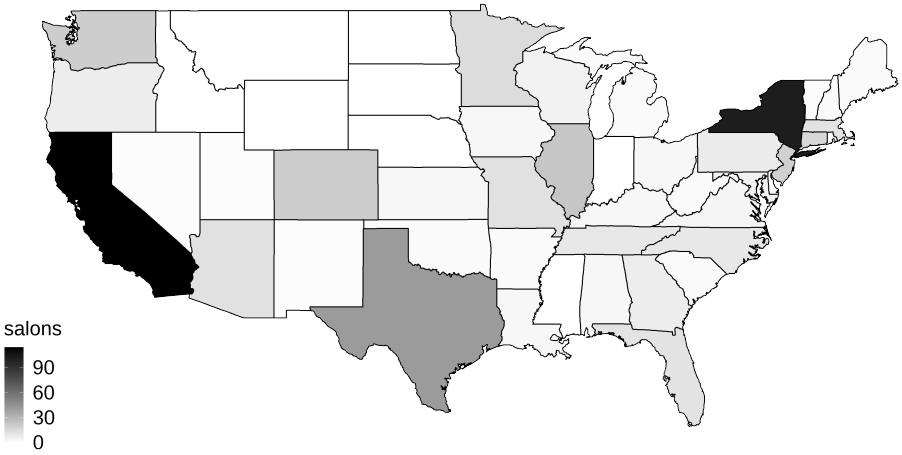
Comments or Questions: kohlhepp@unc.edu

# Table of Contents

# Data

- Salon management software company founded in 2016
- Nationwide, but clients are concentrated in NYC and LA.
- Observe 10.8 million assignments of tasks to hair stylists across hundreds of salons from 2016 to 2021 Q2

# Coverage



# A Data Snapshot

| Firm | Salon | App. | Cust. | Task | Staff | Time Stamp | Price | Duration |
|---|---|---|---|---|---|---|---|---|
| 1 | 1A | 123 | Blake | Advanced Cut | Rosy | 3/26/2021 16:15 | 100 | 72 |
| 1 | 1A | 123 | Blake | Full Head - Highlights | Rosy | 3/26/2021 16:15 | 243 | 127 |
| 1 | 1A | 123 | Blake | Treatment Add On (Olaplex) | Rosy | 3/26/2021 16:15 | 39 | 72 |
| 2 | 2A | 9982 | Grace | Women's Cut | Tyler | 3/17/2021 11:00 | 225 | 43 |
| 2 | 2A | 9982 | Grace | Single Process | Ben | 3/17/2021 11:00 | 200 | 77 |

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| 2 | 2A | 9982 | Grace | Single Process | Ben | 3/17/2021 11:00 | 200 | 77 |

- Granular descriptions are categorized into tasks.
- Analyze one representative product (basket of services) per firm-quarter.
- Measure productivity as total revenue per quarter divided by total of all durations (utilized labor).

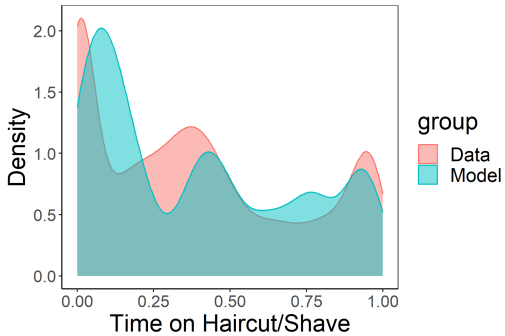
# Wages, Skills, Reference Firm Org. Cost, Etc.

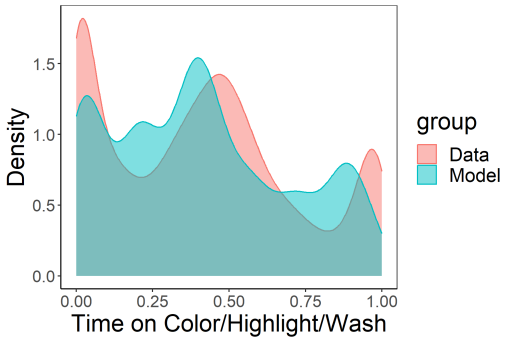
- Demand-side: $log(s_j/s_0) = \sum_{i,k} \theta_i(k) a_j B_j(i,k) - \rho p_j + \nu_j$
- Supply-side: $p_j = \frac{1}{\rho(1-s_j)} + \gamma_1 \tilde{\gamma}_j a_j l_j + w \cdot a_j \cdot E_j + c \cdot \alpha_j + \omega_j$
- Use relative org. costs $\tilde{\gamma}_j a_j l_j$ as instrument for price in demand-side.
- Linear GMM with $K^2 + 1$ equations and $K^2 + 1$ unknowns.
- Adjust prices by markup: $p_j - \frac{1}{\rho(1-s_j)} = \gamma_1 \tilde{\gamma}_j a_j l_j + w \cdot a_j \cdot E_j + c \cdot \alpha_j + \omega_j$
- Linear GMM (OLS) with $2K + 1$ equations and $2K + 1$ unknowns.

# Firms that Do Not Perform One task Type

- The prior procedure will not work if one or more task types are not performed.
- But we identified all market parameters and we proved monotonicity of the s-index in $\gamma_j$.
- Therefore we can invert the s-index for these firms and recover $\gamma_j$ (and also the skill sets of their workers).

# Model Validation: The Task Composition of Salon Jobs



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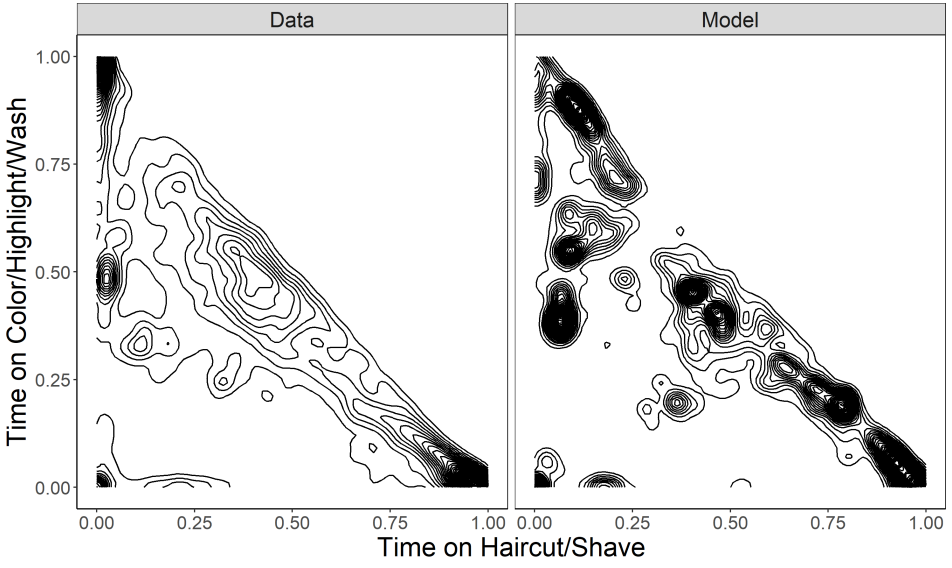

# Simple Example

- Suppose 3 tasks, price sensitivity $\rho = 1$
- 3 worker types with wages $w = (20, 15, 21)$ and skill set:

$$\begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix} = \begin{bmatrix} 23 & 19 & 15 \\ 15 & 15 & 15 \\ 15 & 19 & 26 \end{bmatrix}$$

- Without org. costs, the firm chooses the best person for each task given wages:

$$\begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix} - \rho w = \begin{bmatrix} 3 & -1 & -5 \\ 0 & 0 & 0 \\ -6 & -2 & 5 \end{bmatrix}$$

# Model Validation: The Task Composition of Salon Jobs

| | Task | Variance | Cor. Task 1 | Cor. Task 2 | Cor. Task 3 | Cor. Task 4 | Cor. Task 5 |
|---|---|---|---|---|---|---|---|
| Model | 1 | 0.105 | 1.000 | -0.678 | -0.392 | -0.259 | -0.171 |
| Data | 1 | 0.107 | 1.000 | -0.745 | -0.260 | -0.285 | -0.184 |
| Model | 2 | 0.084 | | 1.000 | -0.154 | -0.164 | -0.156 |
| Data | 2 | 0.094 | | 1.000 | -0.080 | -0.143 | -0.234 |
| Model | 3 | 0.033 | | | 1.000 | -0.013 | -0.077 |
| Data | 3 | 0.014 | | | 1.000 | 0.013 | -0.083 |
| Model | 4 | 0.019 | | | | 1.000 | -0.039 |
| Data | 4 | 0.019 | | | | 1.000 | -0.026 |
| Model | 5 | 0.014 | | | | | 1.000 |
| Data | 5 | 0.021 | | | | | 1.000 |

# Coworker Productivity Spillovers (New York)

| | Skill Set 1 | | | Skill Set 2 | | | Skill Set 3 | | | Skill Set 4 | | | Skill Set 5 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Skill Set | Max. | Med. | Min. | Max. | Med. | Min. | Max. | Med. | Min. | Max. | Med. | Min. | Max. | Med. | Min. |
| 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 2 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 3 | 0.00 | 0.00 | -0.04 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.05 | 0.01 | 0.00 | 0.00 | 0.01 | 0.00 | 0.00 |
| 4 | 0.01 | 0.00 | -0.05 | 0.00 | 0.00 | -0.02 | 0.00 | 0.00 | -0.01 | 0.17 | 0.00 | 0.00 | 0.02 | 0.00 | 0.00 |
| 5 | 0.01 | -0.01 | -0.18 | 0.00 | -0.01 | -0.16 | 0.06 | 0.00 | 0.00 | 0.04 | 0.00 | -0.02 | 0.17 | 0.01 | 0.00 |

# Coworker Productivity Spillovers (New York)

| | Skill Set 1 | | | Skill Set 2 | | | Skill Set 3 | | | Skill Set 4 | | | Skill Set 5 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Skill Set | Max. | Med. | Min. | Max. | Med. | Min. | Max. | Med. | Min. | Max. | Med. | Min. | Max. | Med. | Min. |
| 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 2 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 3 | 0.00 | 0.00 | -0.04 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.05 | 0.01 | 0.00 | 0.00 | 0.01 | 0.00 | 0.00 |
| 4 | 0.01 | 0.00 | -0.05 | 0.00 | 0.00 | -0.02 | 0.00 | 0.00 | -0.01 | 0.17 | 0.00 | 0.00 | 0.02 | 0.00 | 0.00 |
| 5 | 0.01 | -0.01 | -0.18 | 0.00 | -0.01 | -0.16 | 0.06 | 0.00 | 0.00 | 0.04 | 0.00 | -0.02 | 0.17 | 0.01 | 0.00 |

- Own-wage increases tend to increase own productivity (purifying effect).
  - More expensive retained workers are assigned tasks at which they have an advantage.
- Coworker wage increases tend to decrease productivity (sullying effect)
  - As firms layoff workers, coworkers must pick up the slack.
  - The color specialist has to start cutting hair!

# Coworker Productivity Spillovers (New York)

| | Skill Set 1 | | | Skill Set 2 | | | Skill Set 3 | | | Skill Set 4 | | | Skill Set 5 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Skill Set | Max. | Med. | Min. | Max. | Med. | Min. | Max. | Med. | Min. | Max. | Med. | Min. | Max. | Med. | Min. |
| 1 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 2 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| 3 | 0.00 | 0.00 | -0.04 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.05 | 0.01 | 0.00 | 0.00 | 0.01 | 0.00 | 0.00 |
| 4 | 0.01 | 0.00 | -0.05 | 0.00 | 0.00 | -0.02 | 0.00 | 0.00 | -0.01 | 0.17 | 0.00 | 0.00 | 0.02 | 0.00 | 0.00 |
| 5 | 0.01 | -0.01 | -0.18 | 0.00 | -0.01 | -0.16 | 0.06 | 0.00 | 0.00 | 0.04 | 0.00 | -0.02 | 0.17 | 0.01 | 0.00 |

- Own-wage increases tend to increase own productivity (purifying effect).
  - More expensive retained workers are assigned tasks at which they have an advantage.
- Coworker wage increases tend to decrease productivity (sullying effect)
  - As firms layoff workers, coworkers must pick up the slack.
  - The color specialist has to start cutting hair!