The Inner Beauty of Firms

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Motivation: A Tale of Two Salons

Internal Organization: The assignment of workers to tasks within the firm.

Westwood Barber Shop



★ ★ ★ ★ 12/10/2014 - ◆ Updated review

A lovely stylist named Minoo did an incredible job. She colored my hair, freshened up my bob and gave me a great blow dry. The prices are unbelievable. 25 for color. 20 for haircut and 20 for blow dry.



**** 3/10/2019

Throughly enjoyable quality cut from the delightful owners of the salon. At 81 she cut while he cleaned.

John Frieda Salon



***** * * 6/9/2011**

In addition to seeing a different person for your cut and color all the stylists have assistants and they are usually the ones that take you back for washing and drying if your stylist is busy. I've had days where I swear 4-5 people worked on me like I'm a celebrity or something, which speaking of there are often quite a few getting their hair done as well.



★ ★ ★ ★ 1/23/2013

A cut and color here costs more than a monthly payment for some cars.

Source: Yelp.com. Review text truncated for brevity.

Motivation

- ▶ The two salons are organizationally unique.
 - John Frieda is an international brand.
 - Westwood Barber Shop is a local family-owned business.
- The two salons are 4.7 miles apart.
 - They compete for workers in the same labor market.
 - They compete for customers in the same product market.
- ▶ The two salons chose different prices and different internal organizations.

Research Questions

▶ Research Question 1: How do firms choose their internal structure?

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- Research Question 1: How do firms choose their internal structure?
- Research Question 2: What are the implications for product markets, labor markets, and government policy?
- These questions are important:
 - The assignment of workers to tasks is a determinant of productivity.
 - Large literature on specialization across industries/occupations/countries.
 - Much of specialization occurs within the firm via internal organization.

Summary of Paper

► Contribution: An industry equilibrium model of internal organization with organizationally unique firms that can be identified and estimated using task assignment data.

Summary of Paper

- ► Contribution: An industry equilibrium model of internal organization with organizationally unique firms that can be identified and estimated using task assignment data.
- Research Question 1: How do firms choose their internal structure?
 - Answer: Salons choose more complex internal organizations in order to produce higher quality products, but are constrained by firm-specific organization costs.

Summary of Paper

- Contribution: An industry equilibrium model of internal organization with organizationally unique firms that can be identified and estimated using task assignment data.
- Research Question 2: What are the implications for product markets, labor markets, and government policy?
 - Answer: Endogenous and heterogeneous internal organization introduces new economic forces.
 - Example: A minimum wage hike ↑ specialization for minimum wage workers and ↓ specialization for non-minimum wage workers, generating wage spillovers non-monotone in initial wage.
 - Example: A sales tax cut ↑ specialization and worker productivity.

Contribution

An industry equilibrium model of internal organization...

► Task-Based Labor Models. Lazear 2009 (firm-specific task demand); Haanwinckel 2020 (multi-worker firms); Adenbaum 2021 (org. costs); Lindenlaub 2017 (multi-skill workers)

...with organizationally unique firms...

Organizational Economics. Baker, Gibbons, and Murphy 2002 (relational contracts); Garicano and Wu 2012 (knowledge); Meier, Stephenson, and Perkowski 2019 (trust); Martinez et al. 2015 (culture); Alchian and Demsetz 1972, Baker and Hubbard 2003 (monitoring)

...that can be identified and estimated using task assignment data.

- Wage Data. Garicano and Rossi-Hansberg (2006); Caliendo et al. (2012); Garicano and Hubbard (2016)
- Rational Inattention. Jung et al. (2019); Tian (2019); Matêjka and McKay (2015); Lipnowski and Ravid (2022)

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- Salon management software company founded in 2016
- Nationwide, but clients are concentrated in NYC and LA
- Observe 13 million assignments of tasks to hair stylists across hundreds of salons from 2016 to Q3 2021

A Data Snapshot

Firm	Salon	Арр.	Cust.	Task	Staff	Time Stamp	Price	Duration
1	1A	123	Blake	Advanced Cut	Rosy	3/26/2021 16:15	100	72
1	1A	123	Blake	Full Head - Highlights	Rosy	3/26/2021 16:15	243	127
1	1A	123	Blake	Treatment Add On (Olaplex)	Rosy	3/26/2021 16:15	39	72
2	2A	9982	Grace	Women's Cut	Tyler	3/17/2021 11:00	225	43
2	2A	9982	Grace	Single Process	Ben	3/17/2021 11:00	200	77

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- ► Tasks are aggregated to form one representative product per firm-quarter.
- ▶ A firm's **price** is the sum of service prices divided by total customers.
- ▶ A firm's **required labor** is the sum of durations divided by total customers.
- A firm's **task-mix** is the fraction of labor classified as each task.

Creating Task Categories

- 20,560 unique task descriptions.
- A certified cosmetologist was paid to group into 6 categories.
- Two categories merged due to sparsity to yield 5 task categories.

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bond product fusio-dos olaplex barber surcharg color dual doubl
puterbead surjoiding organizations surjoiding confidence of the puterbead organization of the puterbead organization of the puterbead organization o
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Task Categories

Share of Labor	N	Mean	St. Dev.	Min	Pctl(25)	Pctl(75)	Max
Haircut/Shave	4,558	0.41	0.23	0.00	0.26	0.52	1.00
Color/Highlight/Wash	4,558	0.38	0.20	0.00	0.29	0.52	1.00
Blowdry/Etc	4,558	0.09	0.12	0.00	0.03	0.11	1.00
Administrative	4,558	0.05	0.11	0.00	0.002	0.04	1.00
Nail/Etc	4,558	0.06	0.16	0.00	0.00	0.05	1.00

Firm-Quarter Stats.

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What is an Organization Structure?

Definition 1

A firm's organization structure (B_j) , is a matrix where element (i, k) is the fraction of labor assigned to worker i and task k.

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		"Spe	ecialist" S	Salon		"Generalist" Salon					
			Tasks			Tasks					
		Cut	Color	Dry			Cut	Color	Dry		
-E	Α	1/2	0	0	1/2	Α	1/6	1/12	1/12	1/3	Worker
Worker	В	0	1/4	0	1/4	В	1/6	1/12	1/12	1/3	
>	С	0	0	1/4	1/4	С	1/6	1/12	1/12	1/3	Shar
	Tot.	1/2	1/4	1/4		Tot.	1/2	1/4	1/4		re (<i>E</i>)

Task-Mix (α)

What is Organizational Complexity?

Definition 2

The complexity of an organization structure B_j is:

$$I(B_j) = \sum_{i,k} B_j(i,k) log\left(\frac{B_j(i,k)}{\sum_{k'} B_j(i,k') \sum_{i'} B_j(i',k)}\right)$$

► Within-firm specialization Formal Proof Correlation

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- Within-firm specialization Formal Proof Correlation
- Managerial Attention Formal Microfoundation

What is Organizational Complexity?

Definition 2

The complexity of an organization structure B_i is:

$$I(B_j) = \sum_{i,k} B_j(i,k) log\left(\frac{B_j(i,k)}{\sum_{k'} B_j(i,k') \sum_{i'} B_j(i',k)}\right)$$

- ► Within-firm specialization Formal Proof Correlation
- Managerial Attention Formal Microfoundation
- ▶ Instructions (measured in bits) that must be communicated within the firm to implement B_i

Complexity of the Two Structures

		Specialist Salon								
		Cut	Color	Dry						
ee/	Α	1/2	0	0	1/2					
Employee	В	0	1/4	0	1/4					
Em	С	0	0	1/4	1/4					
-	Tot.	1/2	1/4	1/4						

Exactly match tasks and workers

If cut send "0" assign to A

If color send "01" assign to B

If dry send "10" assign to C $\frac{1}{2}(1bit) + \frac{1}{4}(2bit) + \frac{1}{4}(2bit) = 1.5$

Generalist Salon Tasks Color Cut Dry Α 1/6 1/12 1/12 1/3 В 1/12 1/12 1/6 1/3 1/6 1/12 1/12 1/3 Tot. 1/2 1/4 1/4

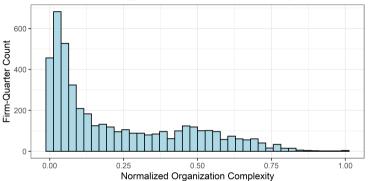
Randomly match tasks and workers

If cut send nothing roll dice

If color send nothing roll dice

If dry send nothing roll dice $\frac{1}{2}(0bit) + \frac{1}{4}(0bit) + \frac{1}{4}(0bit) = 0$

Fact 1: Complexity is heterogeneous and firm-specific.

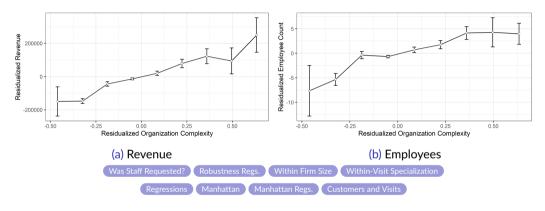


$$I_{j,t} = \bar{I}_j + \bar{I}_t + e_{j,t}$$
 $Var(I_{j,t}) = Var(\bar{I}_j) + Var(\bar{I}_t) + 2Cov(\bar{I}_j, \bar{I}_t) + Var(e_{j,t})$
.0516 .0464 .0002 -.0009 0.0059

Takeaway: Internal complexity is driven by a deep characteristic of the firm.

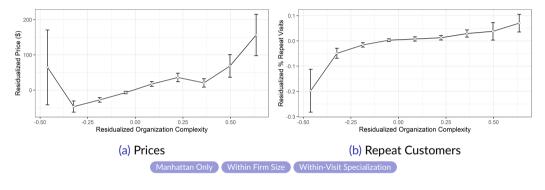
Placebo Simulation

Fact 2: Complex salons have higher revenue and employment



Takeaway: There is an organizational competitive advantage.

Fact 3: Complex salons have higher prices and repeat customers



Takeaway: This advantage operates through quality NOT quantity. Theory

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Model: Salons and Workers

J Salons

- ▶ Salon-specific internal organization cost $\gamma_i \ge 0$
- lacktriangle Leontief task-based production function with task-mix parameter $lpha \in \mathbb{R}_+^K$
 - Producing 1 unit requires assigning α_k labor to task k. Normalize $\sum_k \alpha_k = 1$
 - lacktriangle Homogeneous lpha for exposition only

N Worker Types

- Skill set $\theta_i = \{\theta_{i,1}, ...\theta_{i,k}, ...\theta_{i,K}\}$
- Inelastic total labor supply L_i and wage w_i determined in equilibrium

Model: Salon Choices and Consumers

Salon Choices

- ▶ Org. structure $B_i \in \Delta^{N \times K}$ s.t. $\sum_i B_i(i, k) = \alpha_k$
 - Product Quality: $\xi(B_j) = \sum_{i,k} \theta_{i,k} B_j(i,k)$
 - Per-Unit Wage Bill: $W(B_j) = \sum_{i,k} w_i B_j(i,k)$
 - ▶ Per-Unit Internal Organization Cost: $\gamma_j I(B_j)$ where $I(B_j)$ is complexity
- ▶ Price $p_j \in \mathbb{R}_+$

Consumer Demand D_i

- ▶ Demand depends only on and is strictly increasing in the quality-price index $\xi(B_i) \rho p_i$
 - multinomial logit, nested logit, mixed logit with constant price sensitivity

The Firm's Problem

Denote feasible organization structures $\mathbb{B} = \{B_j \in \Delta^{N \times K} | \sum_i B_j(i, k) = \alpha_k \}$

$$\max_{p_{j},B_{j}\in\mathbb{B}_{j}}\underbrace{D_{j}(\overline{\xi(B_{j})}-\rho p_{j},p_{-j},\xi_{-j})}_{\text{Demand}}\left[p_{j}-\underbrace{\left(\overbrace{\gamma_{j}I(B_{j})}^{\text{org.}}+\overbrace{W(B_{j})}^{\text{avg. wage}}\right)}_{\text{constant marginal cost, }MC_{i}}\right]$$

Equilibrium

An equilibrium consists of firm strategies $\{p_j, B_j\}_{i=1}^J$ and wages w such that:

- 1. Firm strategies maximize profits.
- 2. Labor markets for each worker type clear:

$$\sum_{j} D_{j}(\xi(B_{j}) - \rho p_{j}, p_{-j}, \xi_{-j}) \sum_{k} B_{j}(i, k) = L_{i} \, \forall \, i = 1, ..., N$$

Summary of the Model

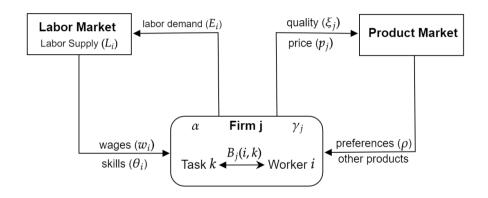


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Characterizing the Firm's Problem

Theorem

An organizational structure (B_i^*) is profit-maximizing if and only if it solves:

$$\min_{B_j \in \mathbb{B}} \gamma_j I(B_j) + W(B_j) - \rho^{-1} \xi(B_j)$$

Characterizing the Firm's Problem

Theorem

An organizational structure (B_j^*) is profit-maximizing if and only if it solves:

$$\min_{B_j \in \mathbb{B}} I(B_j) + \gamma_j^{-1} \sum_{i,k} B_j(i,k) (w_i - \rho^{-1}\theta_{i,k})$$

Rate-distortion problem (information theory)



Characterizing the Firm's Problem

Theorem

An organizational structure (B_i^*) is profit-maximizing if and only if it solves:

$$\max_{B_j \in \mathbb{B}} \sum_{i,k} B_j(i,k) (\rho^{-1}\theta_{i,k} - w_i) - \gamma_j I(B_j)$$

- Rate-distortion problem (information theory)
- Rational inattention problem with MI costs (behavioral econ)
 - Org. frictions make the firm act as if it is run by a manager with limited attention



Characterizing the Firm's Problem

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- Rate-distortion problem (information theory)
- Rational inattention problem with MI costs (behavioral econ)
 - Org. frictions make the firm act as if it is run by a manager with limited attention
- Internal organizations are connected only via wages



The Quality-Wage-Complexity Trade-Off

$$\min_{B_j \in \mathbb{B}} \gamma_j \rho I(B_j) + \rho W(B_j) - \xi(B_j)$$
Quality
$$\xi_j$$

$$V_j$$
Internal Org. Cost
$$V_j$$

$$I_j$$

Organization Frontier

$$\min_{B_j \in \mathbb{B}} \underbrace{I(B_j)}_{\text{complexity}} + \gamma_j^{-1} \left[\underbrace{W(B_j) - \rho^{-1}\xi(B_j)}_{\text{quality-adjusted wages}} \right]$$

Definition

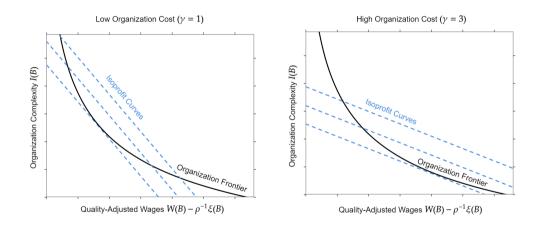
The organization frontier is the set of organization structures which minimize complexity for some quality-adjusted wages.

Because this is a rate-distortion problem:

Proposition

Complexity along the organization frontier is continuous, convex and decreasing in quality-adjusted wages. Proof

Choosing an Organizational Structure



Fitting the Facts

- 1. Fact 1: Complexity is heterogeneous and firm-specific
 - Firms in the same product and labor market choose different internal structures based on individual org. cost (γ_j) .
- 2. Fact 2: Complex salons have more employees and higher revenue
 - In equilibrium complexity and market share are positively correlated.
- 3. Fact 3: Complex salons have higher prices and higher quality
 - Quality is the main benefit of complexity.

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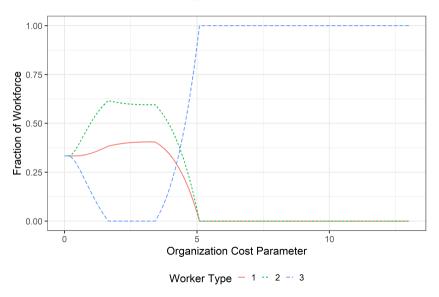
- ▶ 3 tasks with uniform task-mix $\alpha = (1/3, 1/3, 1/3)$, price sensitivity $\rho = 1$
- ▶ 3 worker types with wages w = (21, 20, 15) and skill set:

$$\begin{vmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{vmatrix} = \begin{vmatrix} 15 & 19 & 26 \\ 23 & 19 & 15 \\ 15 & 15 & 15 \end{vmatrix}$$

Wage-adjusted quality:

$$\begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix} - \rho w = \begin{bmatrix} -6 & -2 & 5 \\ 3 & -1 & -5 \\ 0 & 0 & 0 \end{bmatrix}$$

Workforce Composition Heterogeneity



Labor-Labor Substitution Heterogeneity

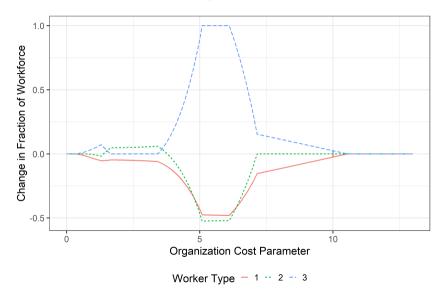


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The Econometric Model

- ▶ Market: Analyze Manhattan 2021 Q2 with sales tax $\tau = 4.5\%$
 - Outside option is no purchase. Compute via Consumer Expenditure Survey.
- ▶ Utility for good $j: u_{z,j} = \xi(B_j) \rho p_j + \epsilon_{z,j}, \epsilon \sim \text{ i.i.d. Type-1 E.V.}$
- ▶ Workers: Base skill β_k , skill gap θ_k
 - ► Color Specialist: $\theta = \{\beta_{cut}, \beta_{color} + \theta_{color}, \beta_{dry}, \beta_{admin}, \beta_{misc}\}$
- Labor Supply: Individual workers also differ in their labor supply
 - 2 workers with same skills may supply different hours
- ► Task Heterogeneity: Different material costs (m)
- Firm Heterogeneity: Firm-specific task-mix (α_j) , effective labor per unit (\bar{a}_j) , exogenous quality (ν_i) , marginal cost shifter (ϕ_i)

The Econometric Model

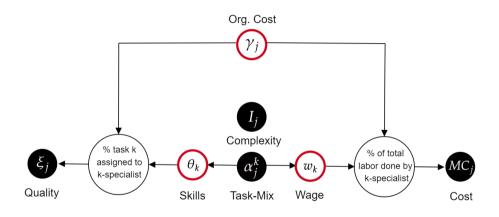
A natural notion of task-specialization:

Definition 3

Task-specialization is the fraction of total labor where a task is assigned to the associated specialist.

- ▶ 1 utility + 5 cost + 5 wages + 10 skills = 21 parameters
- ightharpoonup Call these market parameters and denote Ω
- ▶ 42 salons ⇒ 42 org. cost parameters

Identification Problem



Identification: Firm-Specific Organization Costs γ_j

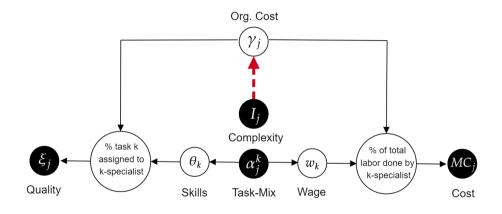
Proposition 1

Organization costs (γ_j) and organization structures (B_j) are a known function of firm task-mixtures (α_j) , complexities (I_j) and market parameters (Ω) for all firms with positive complexity, except for a set of market parameters with measure 0. Visual Intuition

- lacktriangle There is a one-to-one mapping from observed complexity to unobserved γ_j
- $\{\gamma_j\}_{j=1}^J$ do not need to be estimated.
- Instead invert complexity, similar to market share inversion in BLP
- Proof USES an Essential Equilibrium Uniqueness Result

Measure 0 Set

Identification: Firm-Specific Organization Costs γ_j



Identification: Market Parameters

▶ Use firm price FOC (supply side moments):

$$p_{j} = \frac{1}{\rho(1+\tau)(1-s_{i})} + \bar{a}_{j} \left[\gamma(\Omega, l_{j}, \alpha_{j}) l_{j} + wE(\Omega, l_{j}, \alpha_{j}) \right] + m\alpha_{j} + \phi_{j}$$

▶ Use market-share equation (demand side moments):

$$log(s_i) - log(s_0) = \xi(\Omega, I_i, \alpha_i) - \rho(1+\tau)p_i + \beta\alpha_i + \nu_i$$

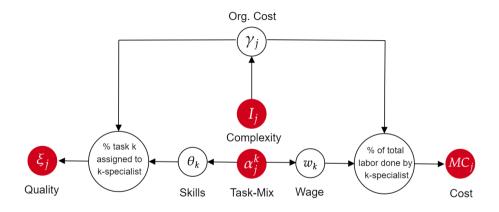
▶ Match avg. beauty salon QCEW wage bill with measurement error:

$$W_i = Ms_i a_i w E(\Omega, I_i, \alpha_i) + e_i$$

 \blacktriangleright The model is globally identified if Ω uniquely satisfies:

$$\mathbb{E}\left[\begin{pmatrix} \phi_j(\Omega, I_j, \alpha_j) \\ \nu_j(\Omega, I_j, \alpha_j) \end{pmatrix} \begin{pmatrix} \alpha_j & \alpha_j I_j \end{pmatrix}\right] = 0 \qquad \mathbb{E}[e_j(\Omega, I_j, \alpha_j)] = 0$$

Heuristic Identification: Market Parameters (Ω)



From Identification to Estimation

- ► Issue
 - \triangleright B_j is a 5 \times 5 matrix which solves a non-linear minimization problem
 - Must solve for B_j repeatedly to obtain γ_j which makes model complexity match observed complexity
 - lacktriangle Must repeat process for each firm and for each set of market parameters Ω
- Solution: Blahut-Arimoto algorithm
 - Fixed point algorithm which is globally convergent
 - Can use because of equivalence to a rate-distortion problem
 - Algorithm

Nested Fixed Point GMM Estimation Routine

Construct the sample analogue of the moment conditions, call it $G(\cdot)$. Then solve:

$$\operatorname*{arg\;min}_{\hat{\Omega}}G(\hat{\Omega})'WG(\hat{\Omega})$$

This amounts to:

- 1. Guess $\hat{\Omega}$.
- 2. Recover implied quality, marginal costs, and organization cost parameters using the Blahut-Arimoto algorithm.
- 3. Evaluate GMM objective. If minimum achieved, stop. Otherwise return to 1.

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Task Parameter Estimates

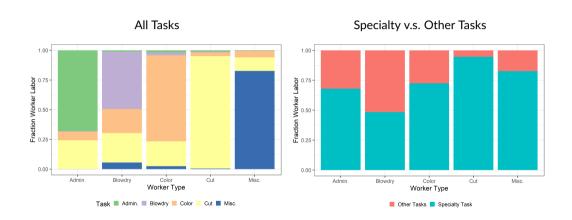
	Associated	d Specialist		
Task	Skill Gap	Wage	Skill Base	Material Cost
Administrative	43.29*	26.99	-16.16	-147.60*
	(21.66)	(63.75)	(14.58)	(13.47)
Blowdry/Etc.	141.69*	20.91	-70.56*	12.39
	(36.67)	(40.22)	(13.57)	(16.65)
Color/Highlight/Wash	60.03*	37.75*	-9.69	56.49*
	(21.24)	(7.00)	(11.97)	(15.79)
Haircut/Shave	32.45*	16.96*		
	(13.07)	(8.32)		
Nail/Spa/Eye/Misc.	66.48	81.16	-252.58*	-1061.12*
	(37.72)	(53.52)	(11.47)	(10.73)

Standard errors from 500 bootstrap replications in parentheses.

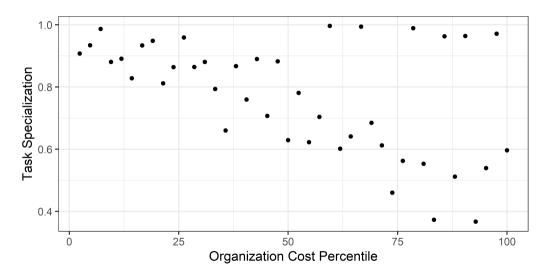


^{*} indicates significance at the 0.05 level.

Equilibrium Task Specialization Across Workers



Equilibrium Task Specialization Across Firms



Cost of Median Complexity Organization Across Firms

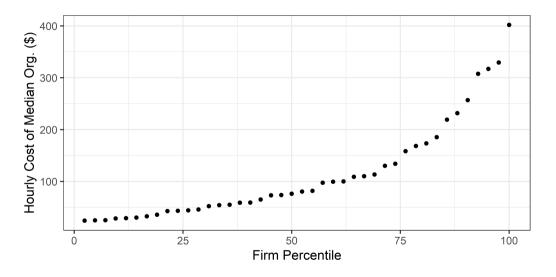


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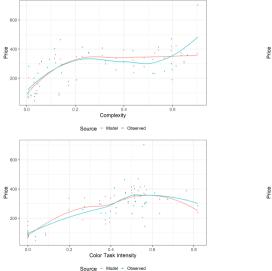
Identification and Estimation

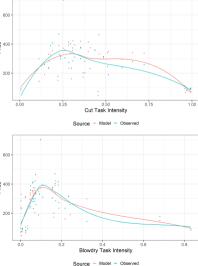
Estimation Results

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Fit: Supply Side Relationships





Validation: The Task Content of Jobs

Model generated jobs:

$$b_{j}(i,k) = \alpha_{k} \frac{\exp(-\gamma^{-1}w_{i} + (\rho\gamma)^{-1}\theta_{i,k})}{\sum_{i'} E_{j}(i')\exp(-\gamma^{-1}w_{i'} + (\rho\gamma)^{-1}\theta_{i',k})}$$

	Total Variance		Between Firm Variance	
Task	Model	Observed	Model	Observed
Haircut/Shave	0.1110	0.1268	0.0597	0.0597
Color/Highlight/Wash	0.1127	0.1105	0.0365	0.0365
Blowdry/Style/Treatment/Extension	0.0472	0.0194	0.0111	0.0111
Administrative	0.0098	0.0080	0.0063	0.0063
Nail/Spa/Eye/Misc.	0.0120	0.0171	0.0050	0.0050



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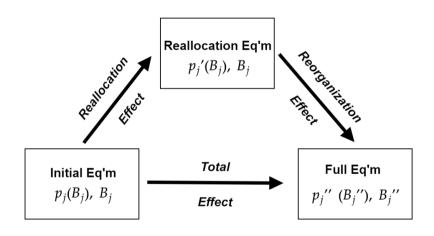
Model Fit and Validation

Counterfactuals

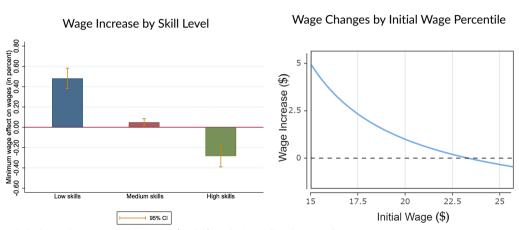
Counterfactual Implementation

- Solving for a new equilibrium
 - 1. Guess wages, solve for organization structures.
 - 2. Use organization structures to obtain costs and qualities.
 - 3. Solve for Nash equilibrium prices.
 - 4. If labor markets clear stop. If not return to step 1.
- ► Compute consumer welfare using closed form

Decomposing Mechanisms



Minimum Wage Increases In Models with Distance Dependent Substitution



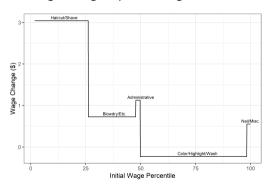
Left is from Gregory and Zierahn (2022), right is stylized example

Minimum Wage Increase from \$15 to \$20

Wages Changes

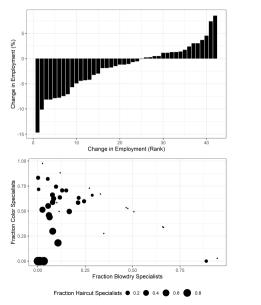
Type	Wage Change	Total Wages Gained/Los
Haircut/Shave - UNEMPLOYED	-100.00%	-\$600,240
Haircut/Shave - EMPLOYED	17.95%	\$1,528,205
Color/Highlight/Wash	-0.61%	-\$228,453
Blowdry/Style/Treatment/Extension	3.48%	\$323,374
Administrative	4.17%	\$47,154
Nail/Spa/Eye/Misc.	0.68%	\$19,319

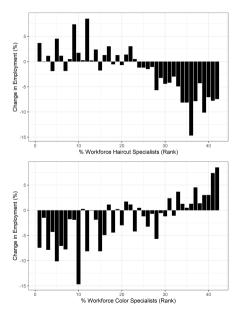
Wage Changes by Initial Wage Percentile



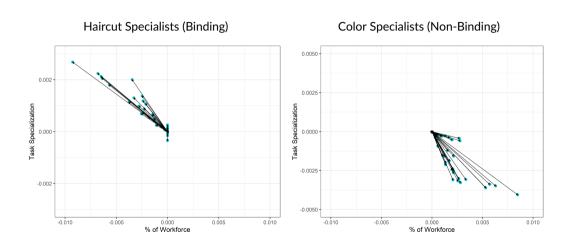
Employment and Wages Technical Details

The Reallocation Effect





The Reorganization Effect



Decomposing Minimum Wage Spillovers

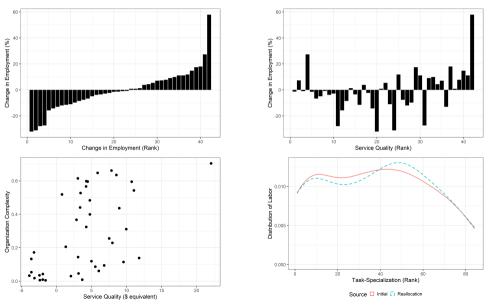
	Reallocation Change		ge		Reorganization Change		
Туре	Employment	Task-Spec.	Wage	Туре	Employment	Task-Spec.	Wage
Haircut/Shave	-5.85%	-0.04%	17.95%	Haircut/Shave	-0.73%	0.12%	0%
Color/Highlight/Wash	0%	-0.17%	-1.13%	Color/Highlight/Wash	0%	-0.33%	0.52%
Blowdry/Style/Treatment/Extension	0%	-0.40%	4.63%	Blowdry/Style/Treatment/Extension	0%	0.03%	-1.15%
Administrative	0%	0.09%	5.22%	Administrative	0%	0.03%	-1.05%
Nail/Spa/Eye/Misc.	0%	-0.03%	0.58%	Nail/Spa/Eye/Misc.	0%	-0.00%	0.10%

Service Sales Tax Elimination (4.5% to 0%)

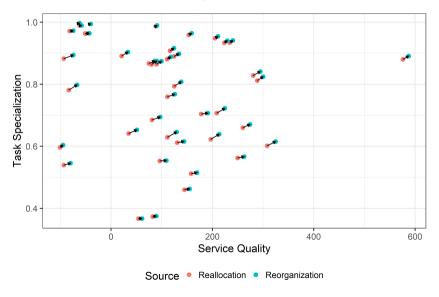
Firm Choi	Firm Choices		Welfare		
Statistic	Total	Source	Change	Percent Change	
Avg. Price	8.68%	Salon Profit	\$942,740	0.58%	
Avg. Complexity	5.53%	Consumer Welfare	-\$494,199	-0.30%	
Avg. Quality	10.03%	Wages	\$11,603,777	7.12%	
Task Specialization	1.83%	Tax Revenue	-\$11,739,300	-7.20%	
		Total Welfare	\$313,017	0.19%	

Effects by Worker Type

Sales Tax Elimination Reallocation Effect



Sales Tax Elimination Reorganization Effect



Conclusion

- ► This paper incorporates firm organizational capabilities into an estimable industry equilibrium model.
- ▶ The model is general and can be easily extended.
 - Multiplicative quality (i.e. Kremer's O-Ring)
 - Quantity-based productivity (i.e. manufacturing)
 - Large firms (continuous tasks, worker types)
- ► Endogenous and heterogeneous internal org ⇒ classic policies have new effects.
- Many new questions:
 - ▶ How does internal organization affect human capital accumulation?
 - How does labor market power impact internal organization?
 - ► How do workers value generalized or specialized jobs?
 - Are economies with specialized firms less resilient?

Table of Contents

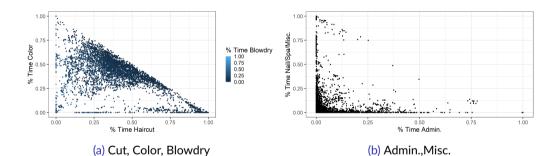
Appendix

Firm-Quarter Statistics

Statistic	N	Mean	St. Dev.	Min	Pctl(25)	Pctl(75)	Max
Revenue	4,558	213,201.30	248,359.90	5	58,912.5	271,236.5	2,559,703
Price	4,558	199.73	135.16	0.20	111.71	261.88	3,180.44
Employees	4,558	13.38	10.79	1	6	17	92
Customers	4,558	1,159.23	1,098.45	1	397	1,619	16,768
Task Categories	4,558	4.45	0.86	1	4	5	5
Labor per. Customer	4,558	2.15	1.63	0.10	1.52	2.57	61.33

Back Task-Mix Variation

Task-Mix Variation



Back

Organization Costs As Average Task-Specialization

Define the generalist job as the job as: $b_i^G(k) = \alpha_k$

Proposition 2

Complexity $(I(B_j))$ is the weighted-average Kullback-Leibler divergence between the jobs at a firm and the firm's generalist job $b_j^G(k)$, where the weights are the share of each worker type.

Proof. Using the definition of mutual information, we can write out complexity as:

$$I(B_{j}) = \sum_{i,k} B(i,k) log \left(\frac{B(i,k)}{\sum_{k'} B(i,k') \sum_{i'} B(i',k)} \right) = \sum_{i,k} E_{i} \frac{B(i,k)}{E_{i}} log \left(\frac{B(i,k)}{E_{i}\alpha_{k}} \right)$$

$$= \sum_{i} E_{i} \sum_{k} b_{i}(k) log \left(\frac{b_{i}(k)}{\alpha_{k}} \right) = \sum_{i} E_{i} \sum_{k} b_{i}(k) log \left(\frac{b_{i}(k)}{b_{j}^{G}(k)} \right)$$

$$= \sum_{i} E_{i} D_{KL}(b_{i}||b_{j}^{G})$$

Managerial Attention

- ightharpoonup X is the task type, with prior α . Y is assigned worker type. Manager's payoff from the assignment of workers to tasks is m(X,Y).
- Manager chooses any signal Z with info about the task type and an assignment function $\delta(Z)$ mapping signal to an assignment.
- Cost of signal is γ_j multiplied by the mutual information between the signal and the task type:

$$\max_{\delta,Z} \mathbb{E}[m(X,\delta(Z))] - \gamma_j I(X,Z)$$

Jung et al. (2019) show it is WLOG to choose joint distribution directly:

$$\max_{B_j \in \mathbb{B}_j} \mathbb{E}[m(X, Y)] - \gamma_j I(X, Y)$$



Revenue Regressed on Complexity

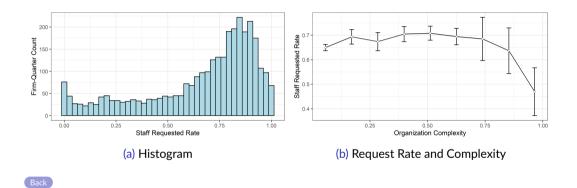
Model:	(1)	(2)	(3)	(4)	(5)	(6)
Organization Complexity	456571.3***	440904.1***	485026.4***	486995.5***	271694.6**	261697**
	(100394.8)	(108427.1)	(116918.9)	(125004.8)	(87031.1)	(80920.6)
Staff Request Rate						-94370.7
						(89112.9)
Task Mix Control				Yes	Yes	Yes
Fixed-effects						
Quarter-Year		Yes	Yes	Yes	Yes	Yes
County			Yes	Yes	Yes	Yes
Firm Size					Yes	Yes
Fit statistics						
Observations	5,116	5,116	5,116	5,116	5,116	5,116
R ²	0.01475	0.01915	0.3104	0.31047	0.34273	0.34365

Clustered standard-errors in parentheses

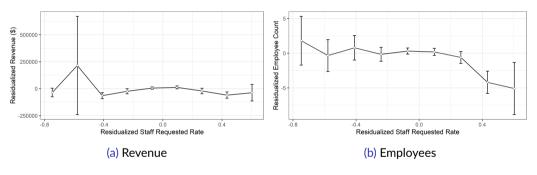
Signif. Codes: ***: 0.001, **: 0.01, *: 0.05



Was Staff Requested?

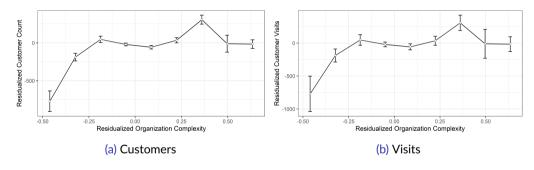


Was Staff Requested?





Relationship Between Complexity and Customers/Visits





Firm Size and Complexity Regressions

Dependent Variables: Model:	Revenue (1)	Employees (2)	Utilized Labor (3)	Customers (4)	Visits (5)
Variables					
Org. Complexity	347549.2***	9.75**	26481	334.6	731.7
	(79546.2)	(3.016)	(35653.2)	(259.6)	(450.1)
Fixed-effects					
Quarter-Year	Yes	Yes	Yes	Yes	Yes
County	Yes	Yes	Yes	Yes	Yes
Fit statistics					
Observations	4,558	4,558	4,558	4,558	4,558
R ²	0.32465	0.34319	0.28918	0.34901	0.35004

Standard-errors clustered at the salon level.

Signif. Codes: ***: 0.001, **: 0.01, *: 0.05



Manhattan Firm Size and Complexity Regressions

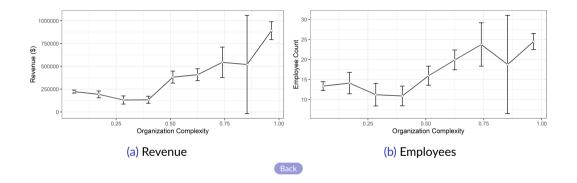
Dependent Variables: Model:	Revenue (1)	Employees (2)	Utilized Labor (3)	Customers (4)	Visits (5)
Variables					
Org. Complexity	430406.6*	12.55	-17733.9	277.2	876.9
	(179977.4)	(6.531)	(70765.2)	(600)	(907.1)
Fixed-effects					
Quarter-Year	Yes	Yes	Yes	Yes	Yes
Fit statistics					
Observations	595	595	595	595	595
R ²	0.33485	0.21039	0.20359	0.44164	0.48831

Clustered standard-errors in parentheses

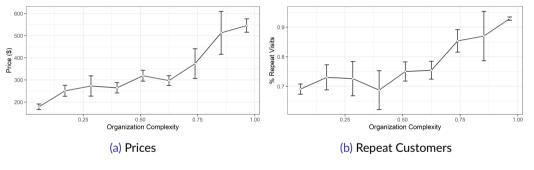
Signif. Codes: ***: 0.001, **: 0.01, *: 0.05



Fact 2: Complex salons have higher revenue and employment



Fact 3: Complex salons have higher prices and repeat customers





Quantity Model Implies Price ↓ Complexity

$$F_{\alpha,B}(a_j) = \min \left\{ \frac{a_1}{\alpha_1 \sum_i \theta_{i,1} B_j(i,1)}, \dots \frac{a_k}{\alpha_k \sum_i \theta_{i,k} B_j(i,k)}, \dots, \frac{a_K}{\alpha_K \sum_i \theta_{i,K} B_j(i,K)} \right\}$$

Given any fixed organizational structure, the efficient way to produce a single unit of output is to set $a_k = \alpha_k \sum_i \theta_{i,k} B_j(i,k)$. Thus marginal costs are constant and consist of the per-unit wage bill and organization costs:

$$MC_j = \sum_i w_i \sum_k \alpha_k \sum_i \theta_{i,k} B_j(i,k) + \gamma_j I(B_j)$$

Proposition 3

Under these assumptions, prices are decreasing with organizational complexity.



Proof of Theorem: Only if Direction 1/2

- ▶ Consider any feasible (p', B'_i) where price is higher than marginal cost.¹
- ▶ There always exists B_i^* which solves the equivalent problem.²
- ► Construct $p_j = p'_j + \gamma_j I(B_j^*) + W(B_j^*) \gamma_j I(B'_j) W(B'_j)$. This price is feasible b/c $p'_j \gamma_j I(B'_j) W(B'_j)$ is price less MC and $\gamma_j I(B_j^*) + W(B_j^*)$ is positive.
- ▶ By construction, price less marginal cost is equal under (p_j, B_i^*) and (p', B_i') .
- ▶ To show profit is higher under (p_j, B_j^*) we need only show demand is higher.

Back

- 1. When p < MC profit is always negative.
- 2. b/c it is an RI problem (convex objective over compact set).

Proof of Theorem: Only If Direction 2/2

To show demand is higher we need only show the quality-price index is higher:

$$= \xi(B_{j}^{*}) - \rho[p_{j}' + \gamma_{j}I(B_{j}^{*}) + W(B_{j}^{*}) - \gamma_{j}I(B_{j}') - W(B_{j}')]$$
(1)
$$= \xi(B_{j}^{*}) - \rho[p_{j}' + \gamma_{j}I(B_{j}^{*}) + W(B_{j}^{*}) - \gamma_{j}I(B_{j}') - W(B_{j}')] + \xi(B_{j}') - \xi(B_{j}')$$
(2)
$$= \xi(B_{j}') - \rho[p_{j}' + \gamma_{j}I(B_{j}^{*}) + W(B_{j}^{*}) - \gamma_{j}I(B_{j}') - W(B_{j}') - \rho^{-1}\xi(B_{j}^{*}) + \rho^{-1}\xi(B_{j}')]$$
(3)
$$= \xi(B_{j}') - \rho p_{j}' - \rho[\gamma_{j}I(B_{j}^{*}) + W(B_{j}^{*}) - \rho^{-1}\xi(B_{j}^{*}) - \{\gamma_{j}I(B_{j}') + W(B_{j}') - \rho^{-1}\xi(B_{j}')\}]$$

$$\leq 0 \text{ because } B_{j}^{*} \text{ minimizes}$$
(4)
$$\geq \xi(B_{j}') - \rho p_{j}'$$
(5)

Back

Proof of Theorem: If Direction

- Suppose there exists B'_j which maximizes profit but does not solve the RI problem.
- ▶ As before, there exists B_i^* which does solve.
- ightharpoonup Construct p_j as before.
- ▶ Because B_j' does not solve the RI problem, we have that $\xi(B_j^*) \rho p_j > \xi(B_j') \rho p_j'$
- ▶ This implies B_i^* does not maximize profit, a contradiction.



Proof of Frontier Shape and Profit/Complexity Relationship 1/2

- ▶ Denote Q as quality-adjusted wages. Denote $I^*(Q)$ as optimal complexity as a function of quality-adjusted wages.
- ▶ RD equivalence $\implies I^*(Q)$ is continuous, convex and decreasing. Also strictly decreasing above some threshold \bar{Q} (Chen, n.d.).
- The firm's choice of quality-adjusted wages solves:

$$V:=\min_{Q}\gamma I^*(Q)+Q$$

lacktriangle Envelope theorem implies the index and thus profit are increasing in γ :

$$\frac{\partial V}{\partial \gamma} = I^*(Q) \ge 0$$



Proof of Frontier Shape and Profit/Complexity Relationship 1/2

Examining the FOC:

Back

$$\frac{dI^{*}(Q) + \gamma^{-1}Q}{dQ} = \frac{dI^{*}(Q)}{dQ} + \gamma^{-1} = 0 \implies \frac{dI^{*}(Q)}{dQ} = -\gamma^{-1}$$

- ightharpoonup Because I^* is decreasing and convex, its derivative is negative and increasing.
- ▶ Therefore Q which solves is increasing in γ .
- lacktriangle Thus profit and complexity will be positively correlated via $\gamma.$

Profit Under the Econometric Model

$$\frac{\exp(\xi(B_j) - \rho(1+\tau)p_j + \beta\alpha_j + \nu_j)}{\sum_{j'} \exp(\xi(B_{j'}) + -\rho(1+\tau)p_{j'} + \beta\alpha_{j'} + \nu_{j'})} \left[p_j - \bar{\mathbf{a}}_j \left(\gamma_j I(B_j) + W(B_j) + m\alpha \right) - \phi_j \right]$$



A Full Solution Method for Optimal Organization Structure

A globally convergent fixed point algorithm to fully solve based on Blahut (1972):

0. Guess some labor demand E^0 . Create matrix V:

$$V_{i,k} = \exp[\gamma^{-1}(\rho^{-1}\theta_{i,k} - W_i)]$$

1. Compute B^t as:

$$B_{i,k}^t = \alpha_k \frac{V_{i,k} E_k^t}{\sum_i E_i^t V_{i,k}}$$

2. Compute E^{t+1} as:

$$E_i^{t+1} = \sum_k B_{i,k}^t$$

3. If converged exit, else return to Step 1 and advance t.



Minimum Wage Counterfactual Details

- ▶ Counterfactuals assume the utility of not getting a service remains fixed.
- Ruling out Multiple Equilibria
 - Assume beforehand which wages bind (i.e. which wages are \$20)
 - O excess labor supply for all types except binding types.
 - Check that assumed binding types have excess labor supply.
 - If yes, count as an equilibria. If not exclude.
- ▶ I do this for all 2⁵ combinations.
- ► This results in only one equilibrium.



Consumer Welfare

Therefore expected utility of consumer *i* has the well-known closed form:

$$V_{i} = \mathbb{E}[\max_{j} \{\xi_{j} - \rho p_{j} + \epsilon_{i,j}\}] = In \left[\sum_{j=1}^{J} \exp\left(\xi_{j} - \rho p_{j}\right)\right] + C$$

where C is Euler's Constant. There are a mass M of consumers, therefore total consumer expected utility is $M \cdot V_i$. We then can denominate this in dollar terms by dividing by the coefficient on price, ρ . Our measure of total consumer welfare in dollar terms is:

$$CS = \frac{M}{\rho} \left\{ ln \left[\sum_{j=1}^{J} exp \left(\xi_j - \rho p_j \right) \right] + C \right\}$$

With a sales tax τ , it is:

$$CS = \frac{M}{\rho} \left\{ ln \left[\sum_{j=1}^{J} exp \left(\xi_{j} - \rho (1+\tau) p_{j} \right) \right] + C \right\}$$

Equilibrium Uniqueness

Proposition 4

Suppose wages are fixed parameters. A pure strategy equilibrium always exists, and it is unique except over a set of parameters with measure 0.

Proof Sketch:

Bertrand oligopoly with logit demand has unique NE

Caplin and Nalebuff (1991)

Profit is strictly incr. in quality-adjusted (QA) cost

Main Characterization

QA wages and org. costs with mult. equilibria is null

Lipnowski and Ravid (2022)

- ▶ Union of countable null sets (all combinations of *J* org. costs) is null
- ▶ QA wages are function F of params; Jacobian of F is rank $N \times K \implies$ parameters which generate mult. equilibria are measure 0.



Identification Proof Sketch 1/2

- ► Task assignments over worker identities (\tilde{B}_j) are observed. Task assignments over worker types (B_j) are not
- ▶ Lemma: $I(\tilde{B}_j) = I(B_j)$
 - Workers w/ same skill set assigned same tasks

Distraction-Free Property (Tian 2019)

- Then apply data processing inequality or algebra
- ▶ Denote model-generated complexity as $\tilde{I}(\Omega, \gamma_j, \alpha_j)$
- $ightharpoonup \tilde{I}(\Omega, \gamma_j, \alpha_j)$ is a known function

RD Equivalence (Blahut 1972)

 $lacklossim ilde{I}(\Omega,\gamma_j,lpha_j)$ can be arbitrarily well approximated by the Blahut-Arimoto algorithm



Identification Proof Sketch 2/2

▶ Define $Q_i := W(B_i) - \rho^{-1}\xi(B_i)$. By RD equivalence:

$$V:=\min_{B_j\in\mathbb{B}_j}\gamma_jI(B_j)+W(B_j)-\rho^{-1}\xi(B_j)=\min_{Q_j\in\mathbb{Q}_j}\gamma_jI_j^*(Q_j)+Q_j$$

where I_j^* is a decreasing, convex function. The FOC $\frac{dV}{dQ_j} = \gamma_j \frac{dI_j^*(Q_j)}{dQ_j} + 1 = 0$ and convexity imply Q_i is increasing in γ_i .

- ▶ I_j^* is decreasing in Q_j when $I_j^* > 0$ thus decreasing in γ_j .
- ▶ $I_j^*(B_j) = \tilde{I}(\Omega, \gamma_j, \alpha_j) \implies \frac{\partial \tilde{I}(\Omega, \gamma_j, \alpha_j)}{\partial \gamma_j} < 0 \implies \gamma_j$ is identified. Similar to BLP, can recover γ_j by inversion: $\tilde{I}(\Omega, \gamma_j, \alpha_j) = I(\tilde{B}_j)$
- ▶ $\{B_j\}_{j=1}^J$ unique except over a set with measure 0



A Sufficient Condition for the Uniqueness of B_j

Assumption

Define the wage-quality vector of a worker of type i at firm j as $v_{i,j} = \{exp(\gamma_j^{-1}(\rho^{-1}\theta_{i,k} - w_i))\}_{k=1}^K$. Each firm's wage-quality vector $\{v_{i,j}\}_{i\in\mathcal{I}}$ is affinely independent.

Source: Matêjka and McKay (2015)

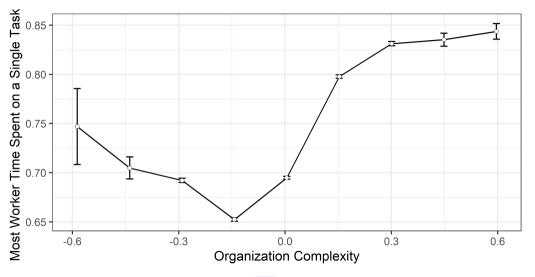


Minimum Wage Counterfactual Employment and Wages

	Initial		Reallocation			Counterfactual	
Worker Type	Hours	Wage	Hours	Wage	Hours	Wage	
Haircut/Shave	537550	\$16.96	506090	\$20.00	502152	\$20.00	
Color/Highlight/Wash	997053	\$37.75	997053	\$37.33	997053	\$37.52	
Blowdry/Style/Treatment/Extension	444040	\$20.91	444040	\$21.88	444040	\$21.64	
Administrative	41860	\$26.99	41860	\$28.40	41860	\$28.12	
Nail/Spa/Eye/Misc.	34844	\$81.16	34844	\$81.63	34844	\$81.71	

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Regressions of Worker Specialization on Organization Complexity



Why Aggregation?

- A single product allows me to focus on the effects of organization on overall salon quality.
- Consumers buy a bundle of services at salons.
- It buys significant numerical/theoretical tractability.
- Nocke and Schutz (2018): any pricing game with multi-product firms and MNL demand can be represented as a single product firm game with transformed qualities and costs:

$$ilde{q}_j =
ho logigg(\sum_k exp((q_k-c_k)/
ho)igg) + 1 \qquad ilde{c}_j = 1$$

Sales Tax Elimination Effects by Worker Type

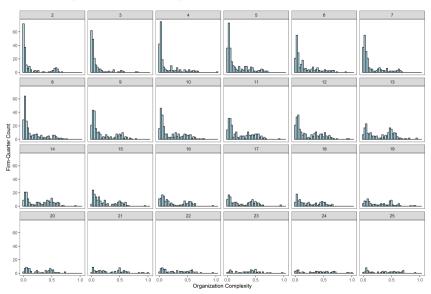
Туре	Wage Change	Task-Spec. Change	
Haircut/Shave	31.99%	0.29%	
Color/Highlight/Wash	20.09%	2.57%	
Blowdry/Style/Treatment/Extension	6.06%	3.01%	
Administrative	17.99%	1.03%	
Nail/Spa/Eye/Misc.	12.74%	2.39%	



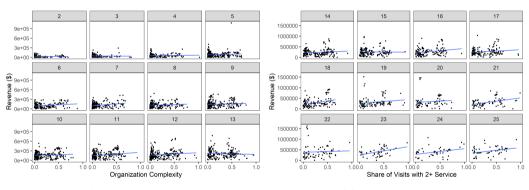
Minimum Wage Welfare Effects

Source	Change	Percent Change	
Salon Profit	-\$714,413	-0.472%	
Consumer Welfare	-\$2,528,784	-1.671%	
Employed Wages	\$1,689,600	1.116%	
Unemployed Wages	-\$600,240	-0.397%	
Total Welfare	-\$2,153,838	-1.423%	

Complexity Histograms Among Similar Size Firms



Revenue and Complexity Among Similar Size Firms

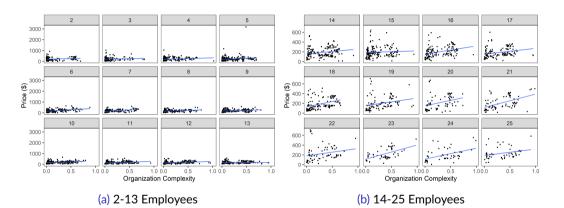


(a) 2-13 Employees

(b) 14-25 Employees

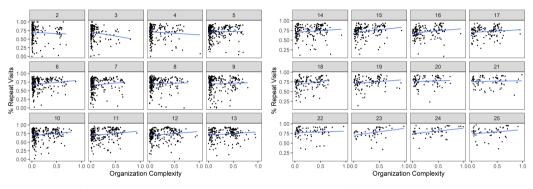


Price and Complexity Among Similar Size Firms





Repeat Visits and Complexity Among Similar Size Firms



(a) 2-13 Employees

(b) 14-25 Employees

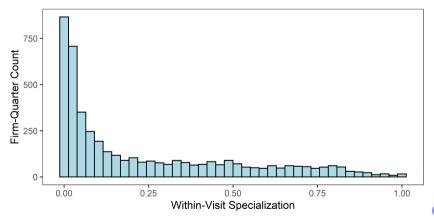


Within-Visit Specialization

- ▶ Within-visit specialization: the number of customer visits³ with two or more employees assigned divided by the number of customer visits with two or more services performed.
- R-squared of complexity regressed on within-visit specialization is 0.5
- ► Two firm-quarters are drawn randomly their ordering according to complexity and within-visit specialization will be the same 74.4% of the time.

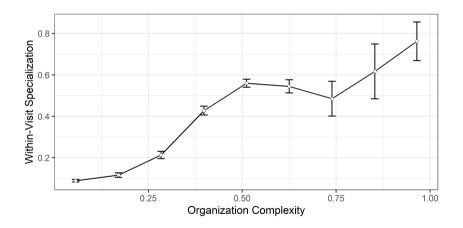
3. Visits are the number of unique customer-date pairs in a quarter.

Within-Visit Specialization Histogram

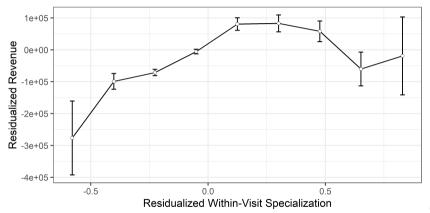




Within-Visit Specialization and Complexity

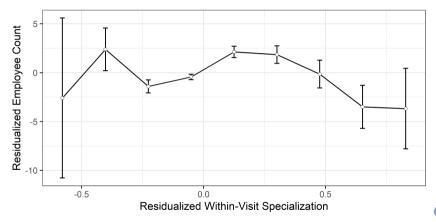


Within-Visit Specialization and Revenue



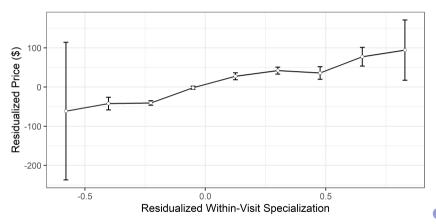


Within-Visit Specialization and Employees



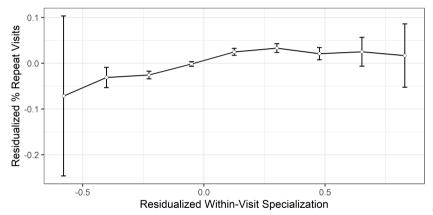


Within-Visit Specialization and Price





Within-Visit Specialization and Repeat Visits





Estimated Organization Structures

		Task					
		Cut	Color	Blow Dry	Admin.	Nail/Misc.	Total
ist	Cut	0.15	0.01	0.001	0.06	0	0.22
	Color	0	0	0	0	0	0
Specialist	Blow Dry	0	0	0	0	0	0
Sp	Admin.	0.31	0.03	0.003	0.45	0	0.784
	Nail/Misc.	0	0	0	0	0	0
	Tot.	0.455	0.036	0.004	0.505	0	1

		Task					
		Cut	Color	Blow Dry	Admin.	Nail/Misc.	Total
	Cut	0.180	0.003	0	0.006	0.003	0.193
ist	Color	0.057	0.553	0	0.016	0.009	0.116
Specialist	Blow Dry	0.012	0.002	0.097	0.003	0.002	0.636
Sp	Admin.	0	0	0	0	0	0
	Nail/Misc.	0.004	0.001	0	0.001	0.050	0.055
	Tot.	0.253	0.559	0.097	0.026	0.064	1

(a) Salon 1, $I_j = 0.03$

(b) Salon 2, $I_j = 0.70$



Variation in Job Task Content

Across Firms

Across Quarters

		Share of Variance				Share of Variance	
Task	Share of Labor	Firm	Within-Firm	Task	Share of Labor	Quarter	Within-Quarter
Haircut/Shave	0.4049	0.3744	0.6256	Haircut/Shave	0.4049	0.0057	0.9943
Color/Highlight/Wash	0.3902	0.2899	0.7101	Color/Highlight/Wash	0.3902	0.0062	0.9938
Blowdry/Style/Treatment/Extension	0.0850	0.5056	0.4944	Blowdry/Style/Treatment/Extension	0.0850	0.0111	0.9889
Administrative	0.0590	0.4900	0.5100	Administrative	0.0590	0.0193	0.9807
Nail/Spa/Eye/Misc.	0.0610	0.4124	0.5876	Nail/Spa/Eye/Misc.	0.0610	0.0118	0.9882



Simulated Complexity with Random Org. Structure

